Interval exchange transformations from tiling billiards

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Joint work with:

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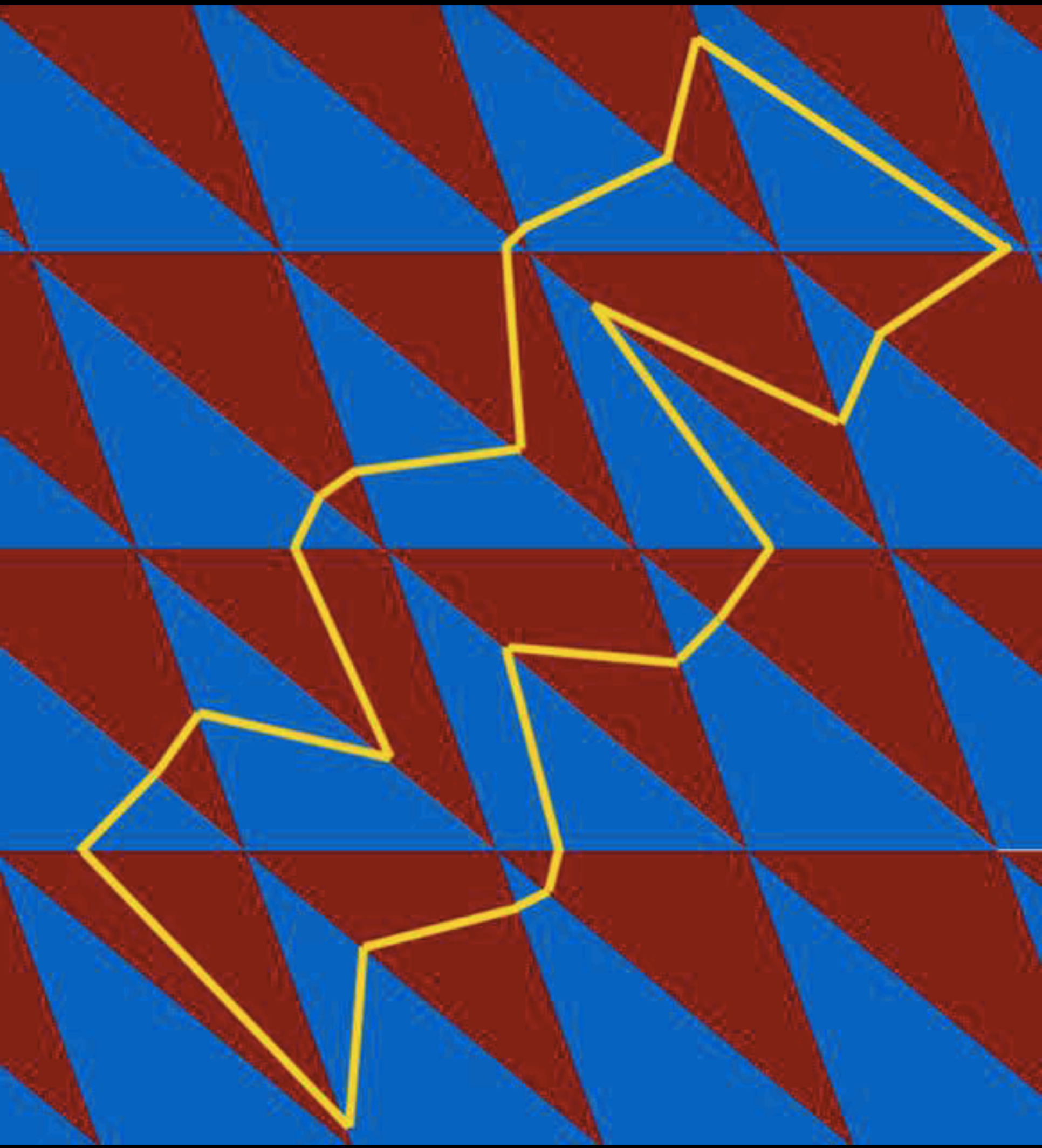
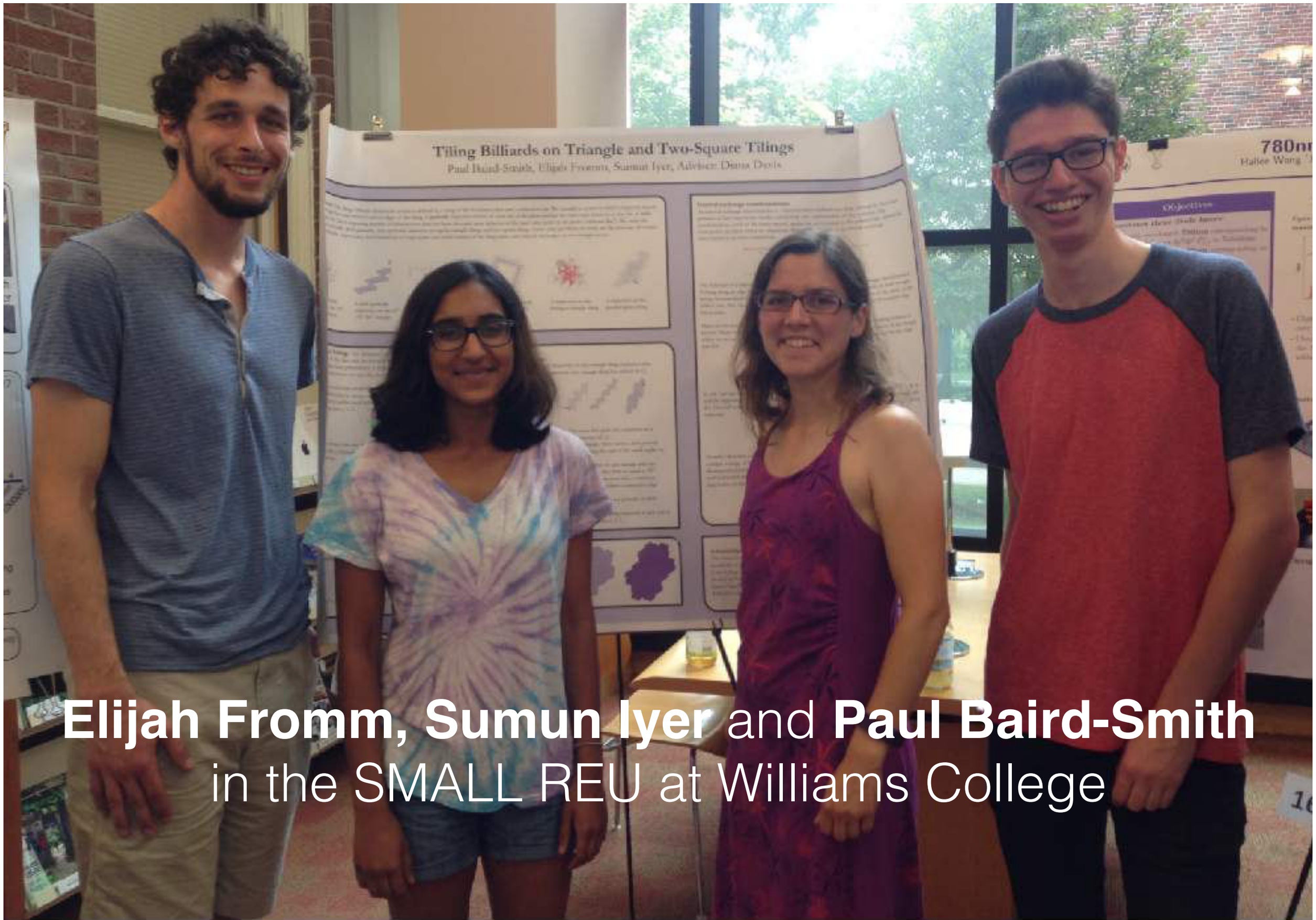
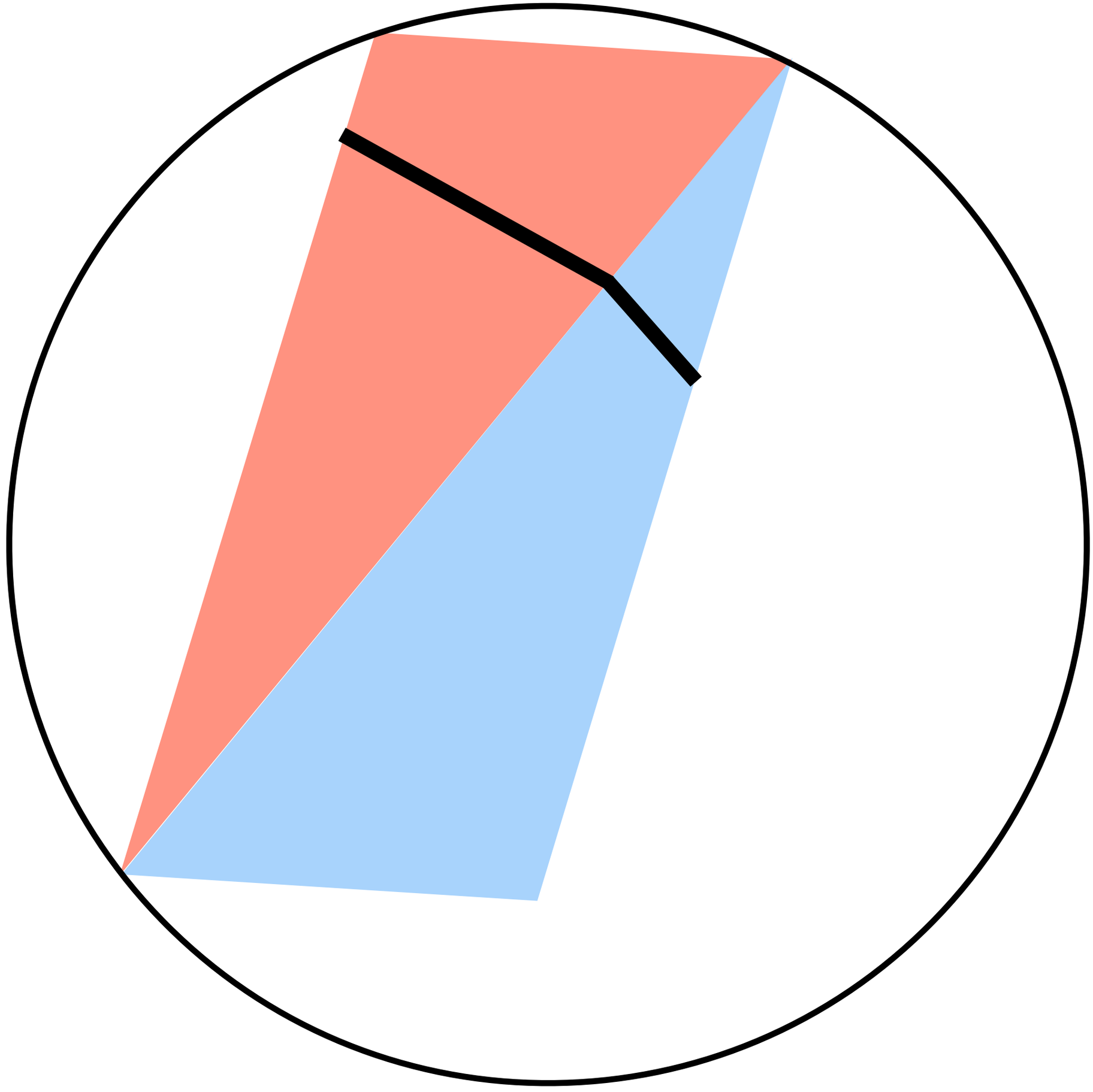
Alex St Laurent
Paul Baird-Smith
Elijah Fromm
Sumun Iyer
Pat Hooper

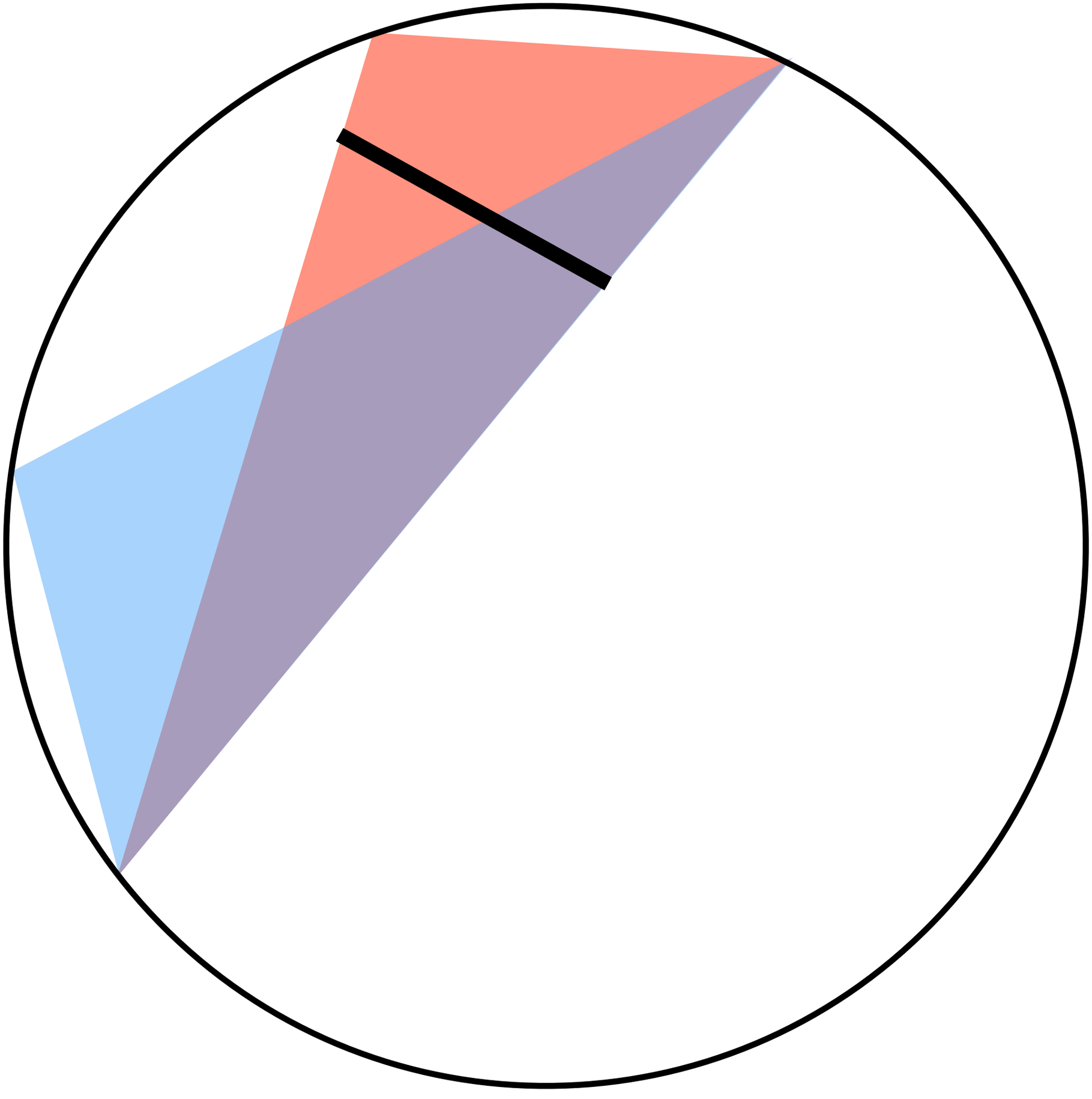


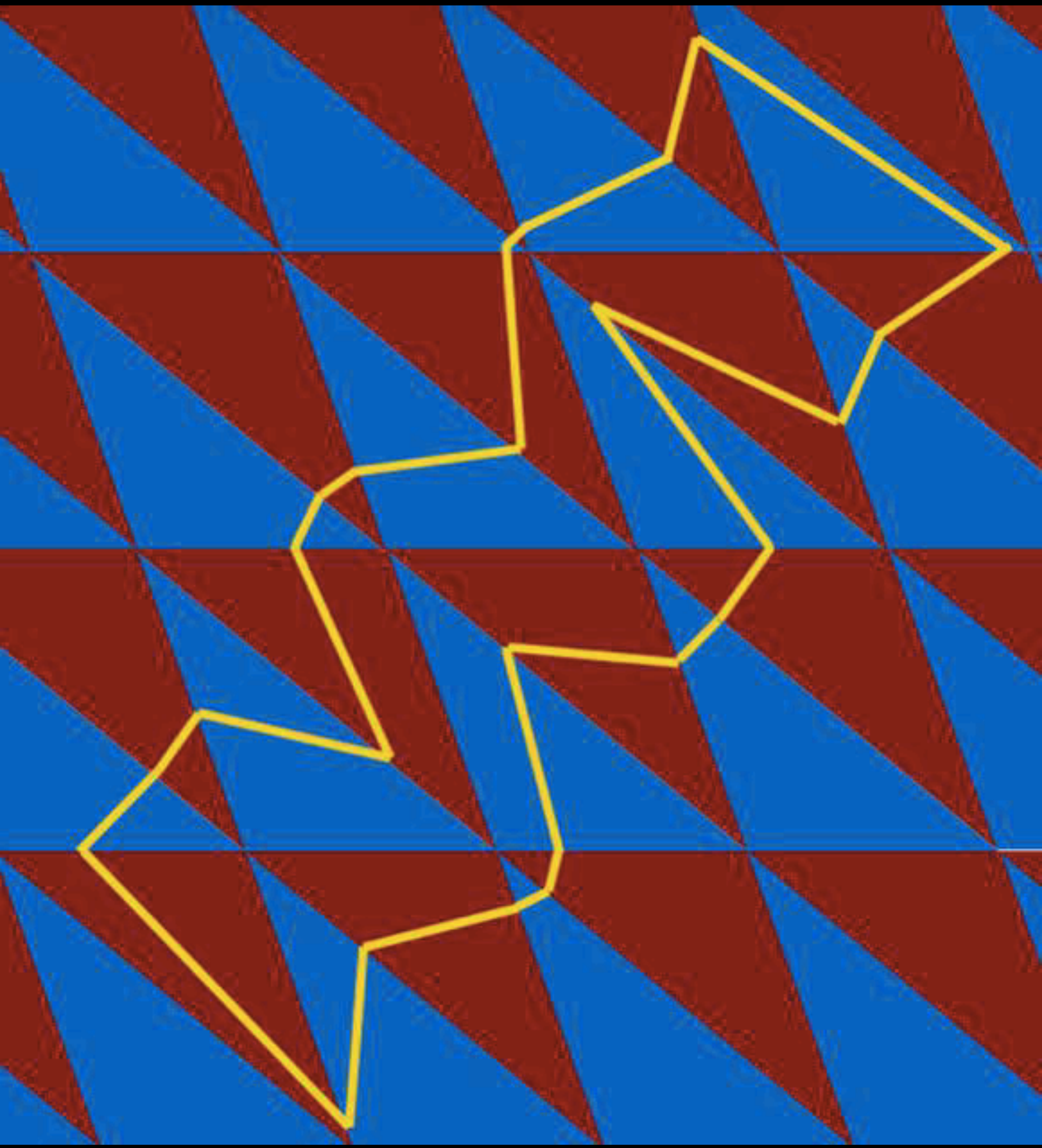
Triangle tilings collaborators



Elijah Fromm, Sumun Iyer and **Paul Baird-Smith**
in the SMALL REU at Williams College

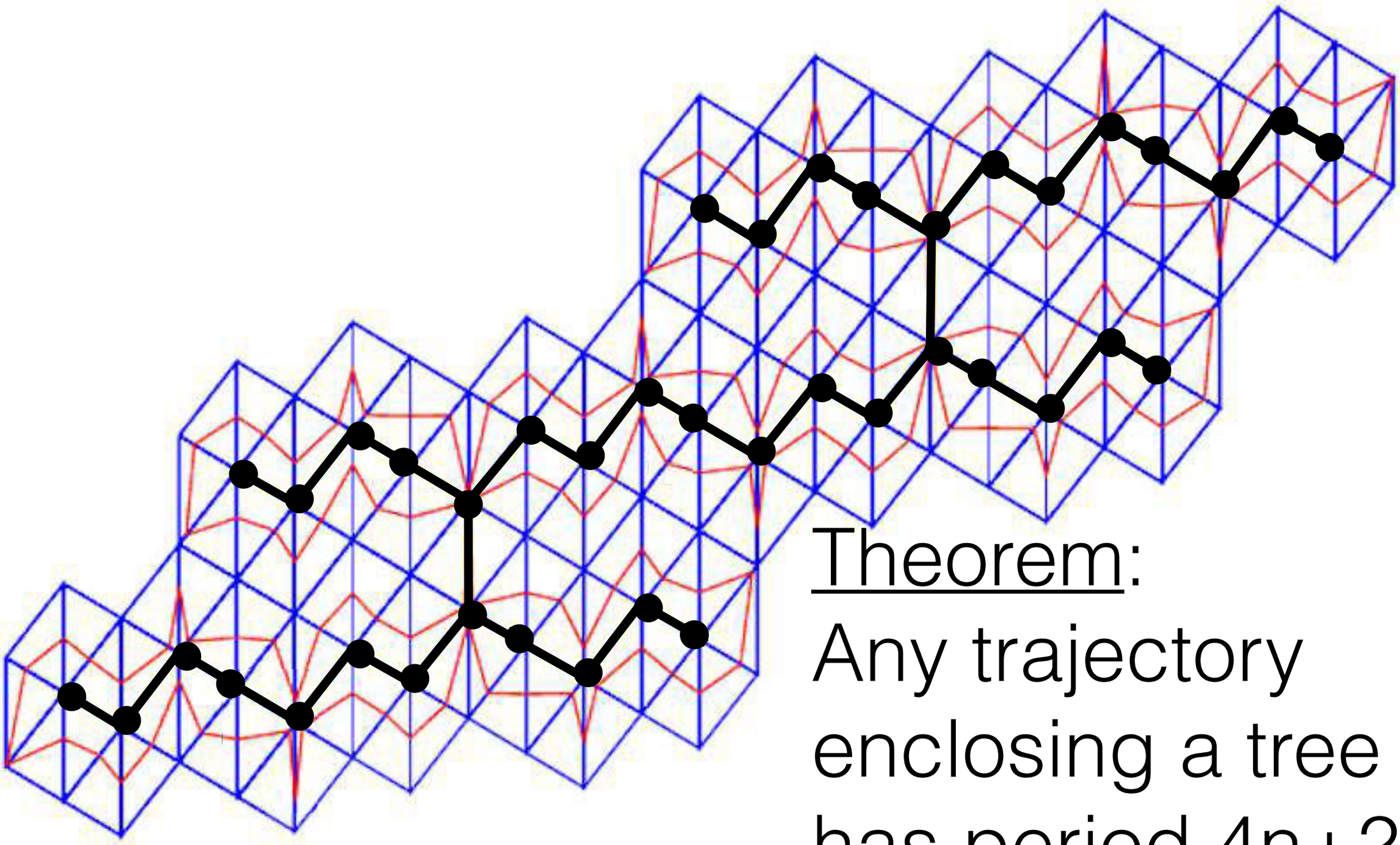






Conjecture:

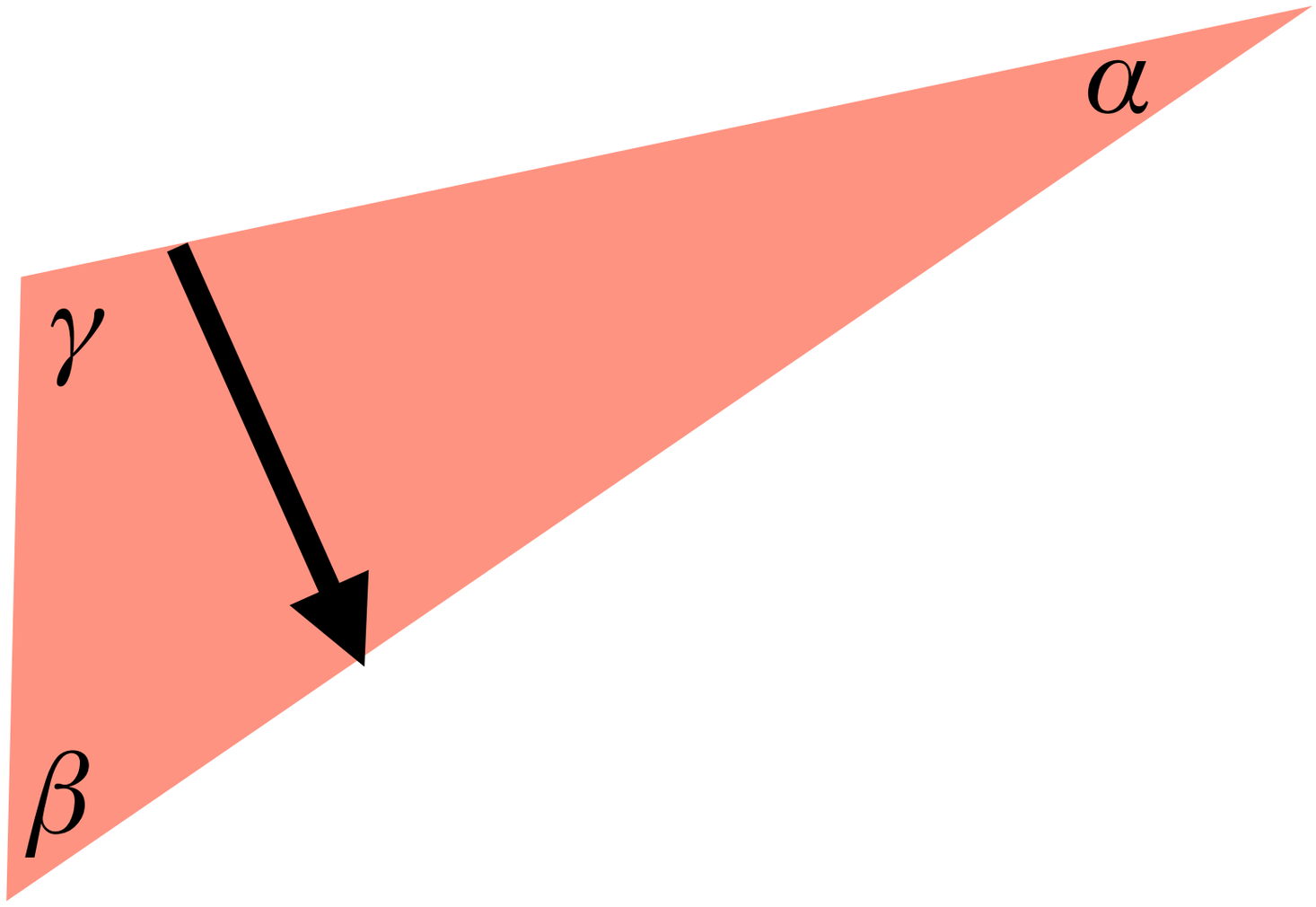
Enclosed vertices & edges form a tree



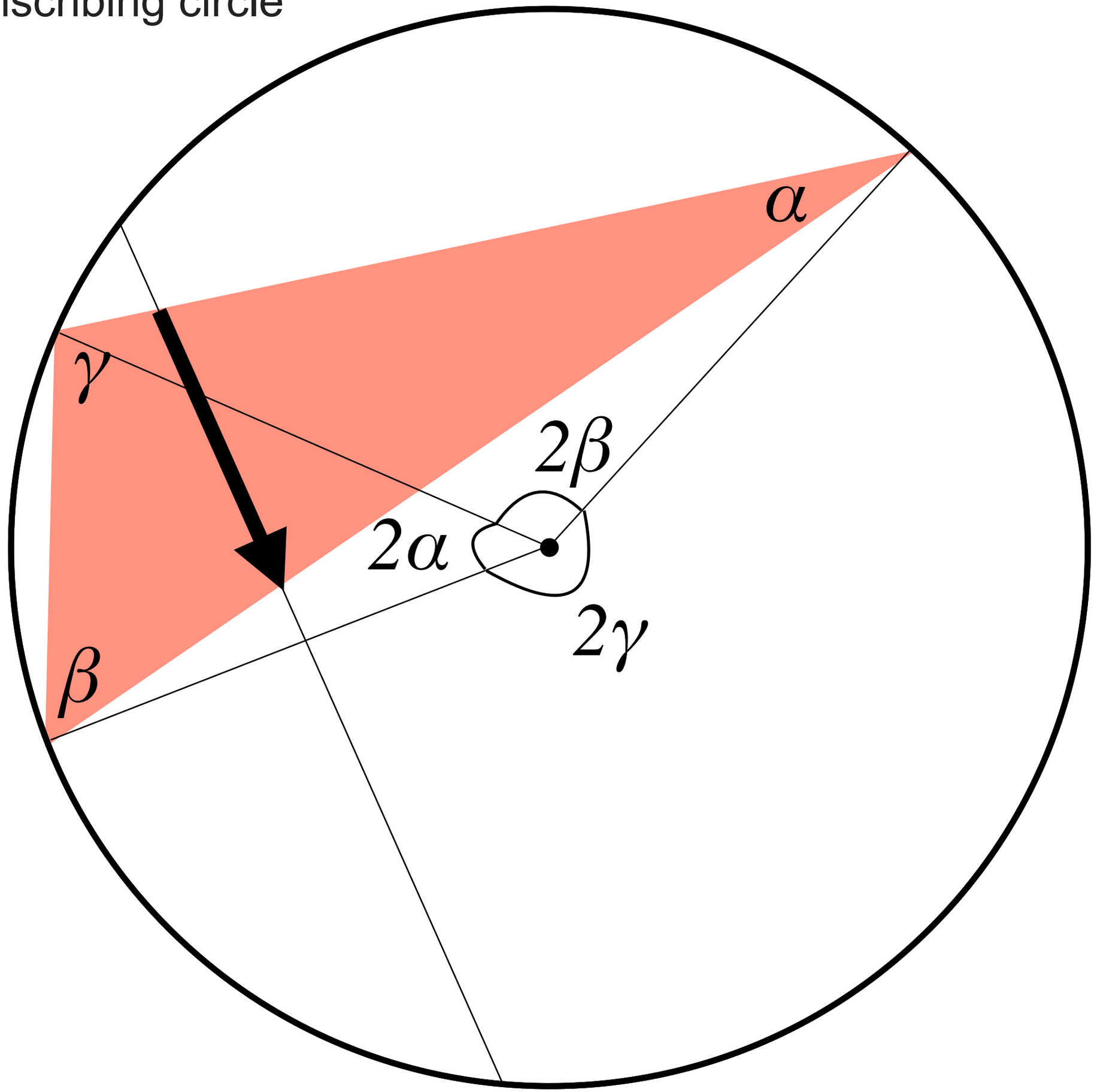
Theorem:

Any trajectory enclosing a tree has period 4n+2

Start with triangle and trajectory

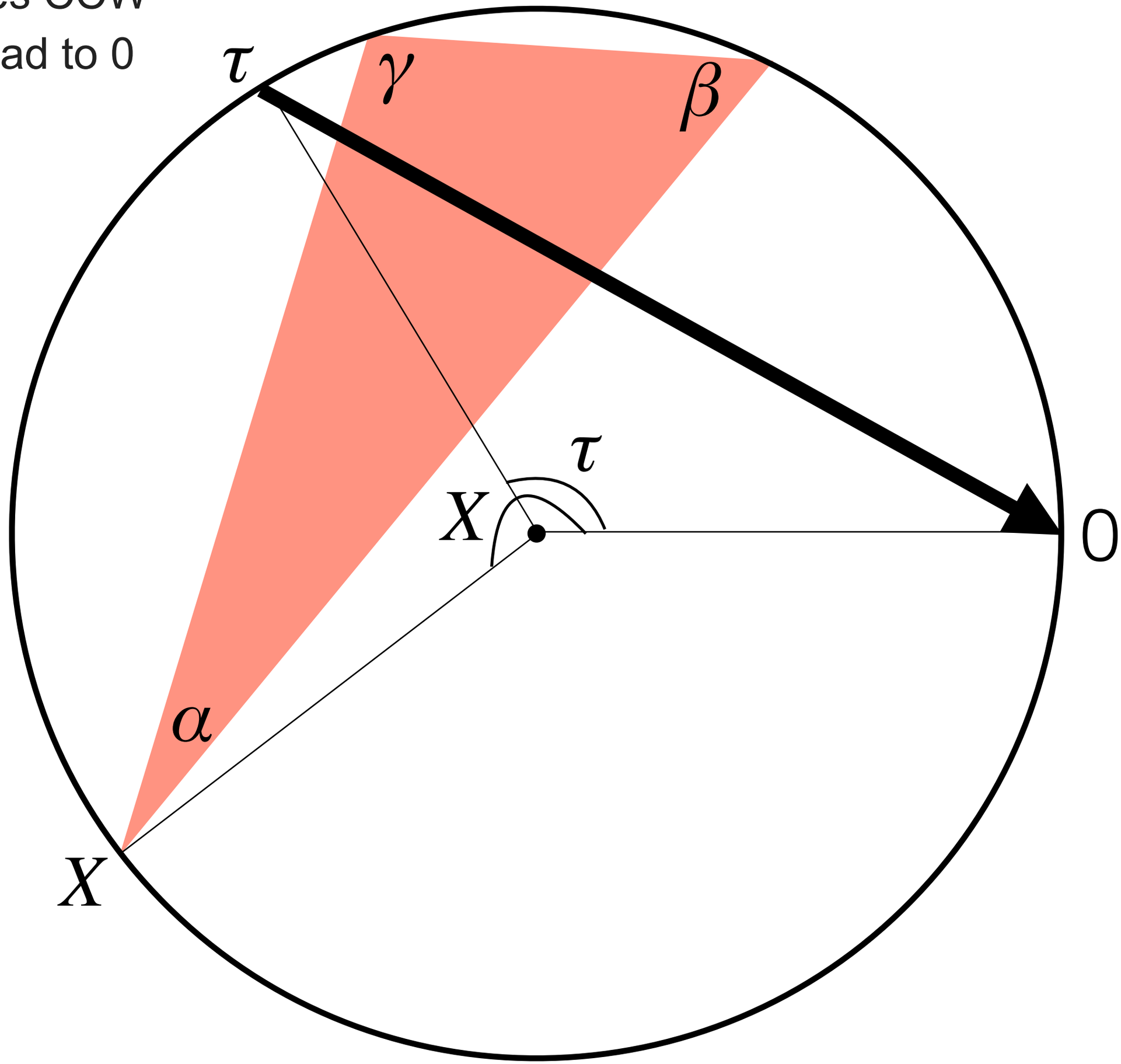


Add circumscribing circle



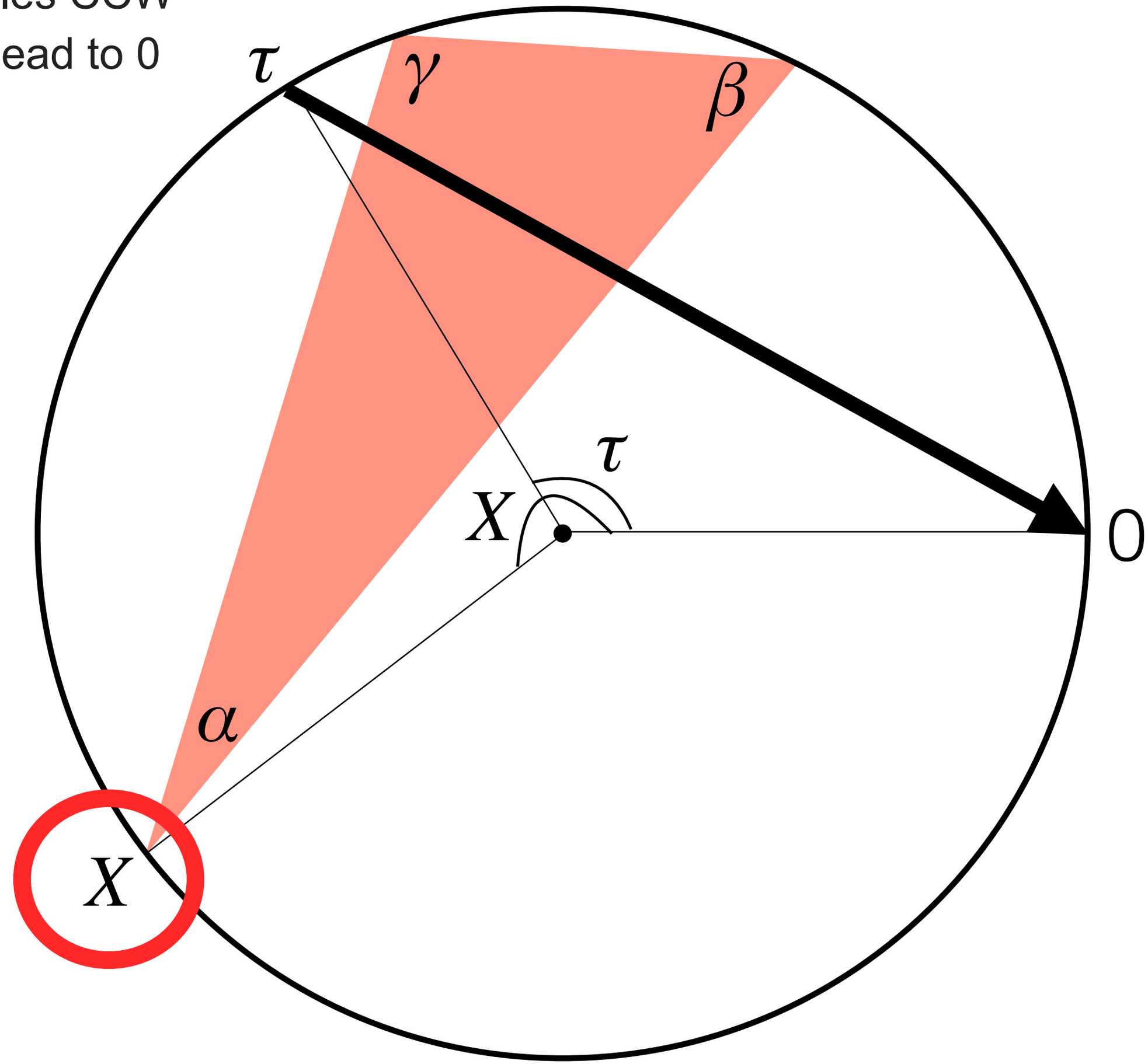
Flip: angles CCW

Rotate: head to 0

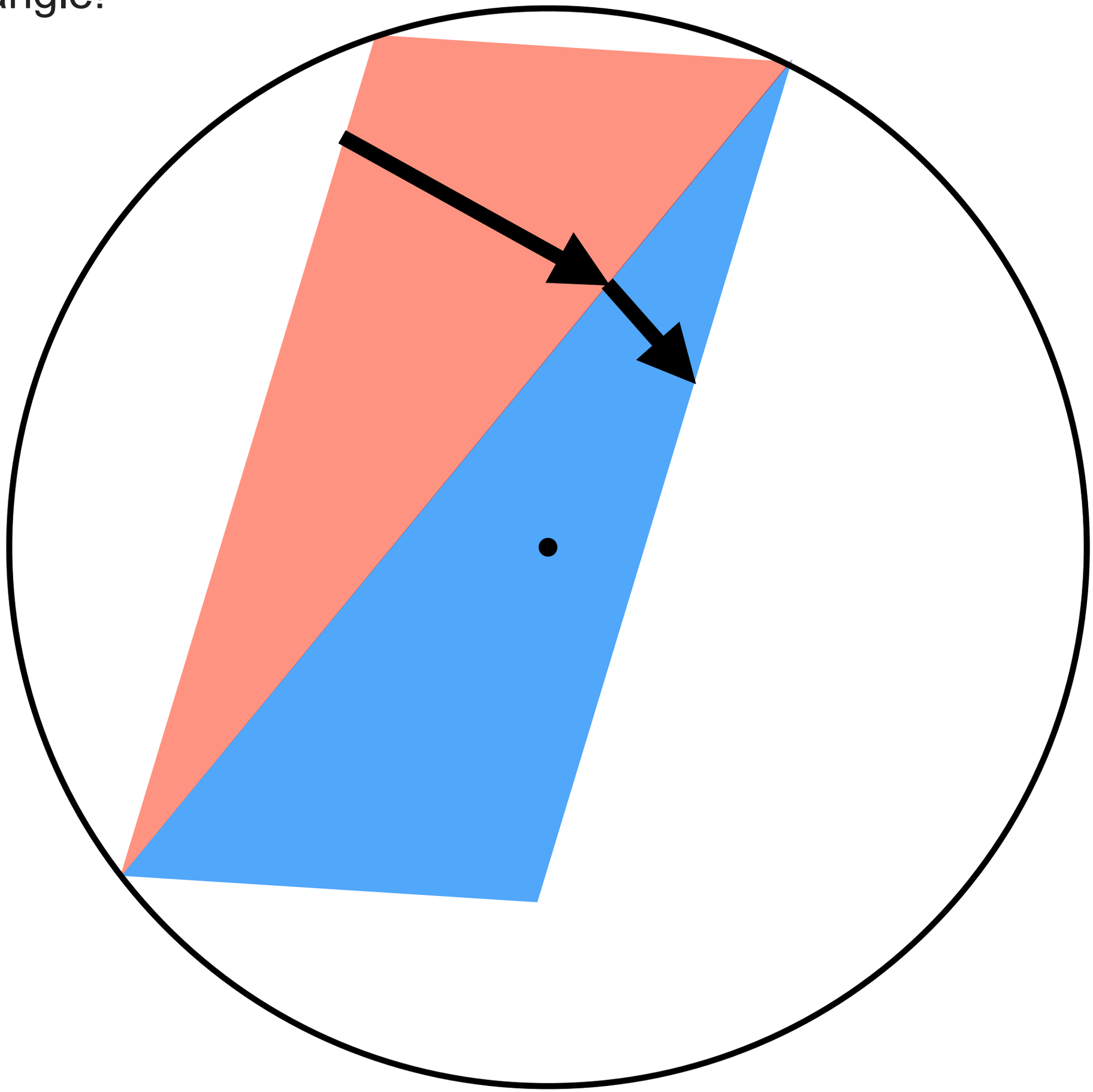


Flip: angles CCW

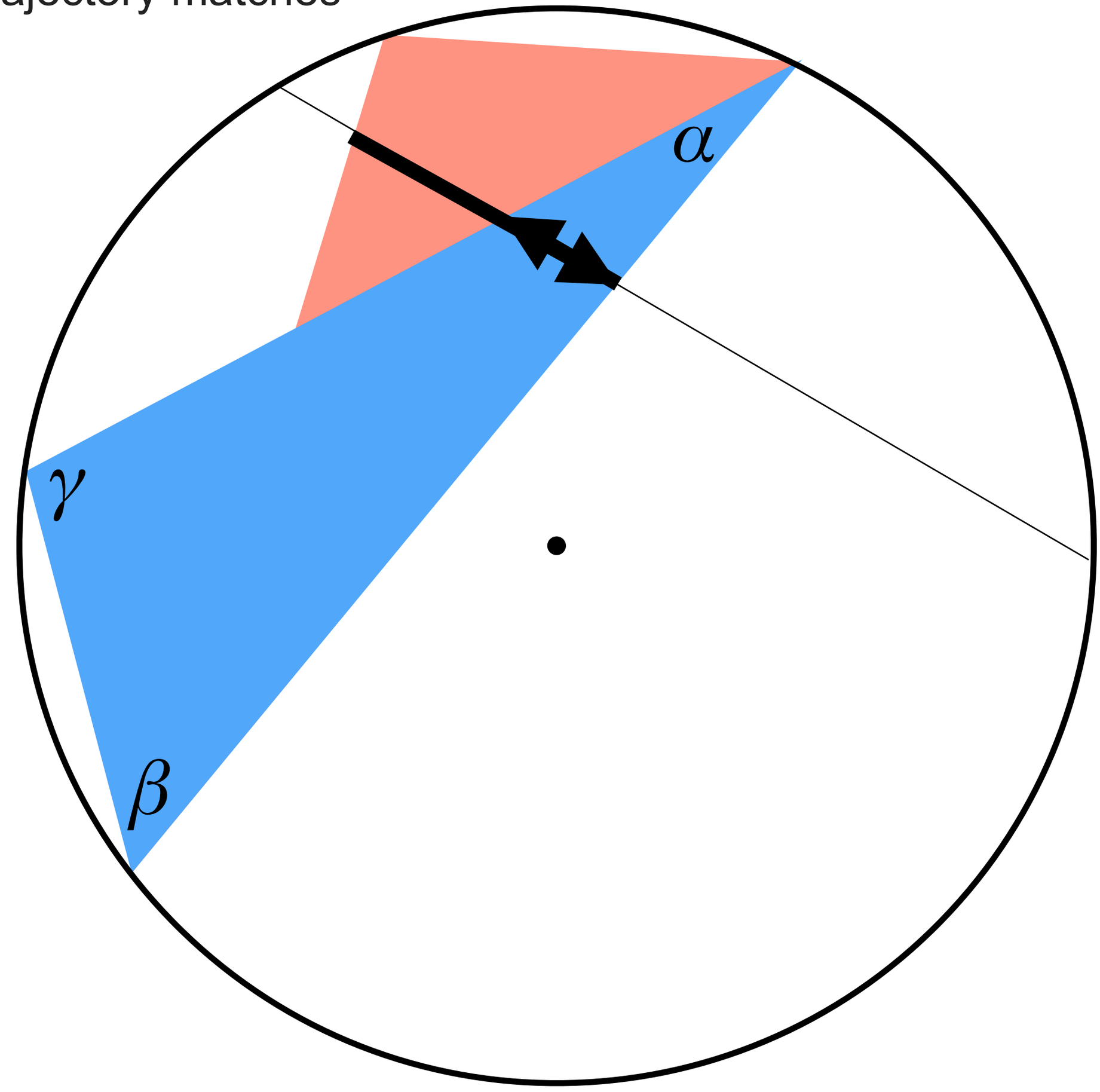
Rotate: head to 0



Next triangle!

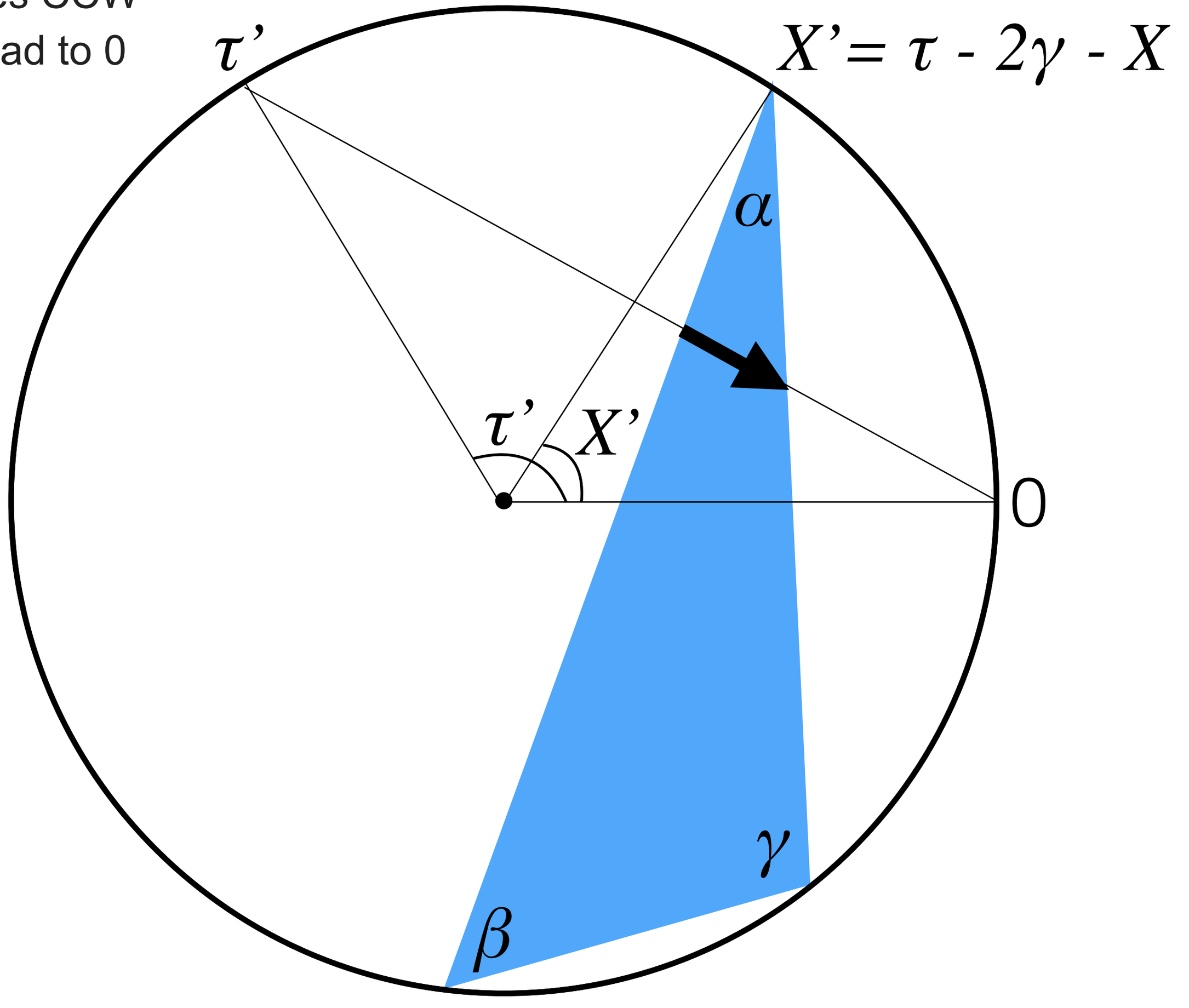


Fold so trajectory matches



Flip: angles CCW

Rotate: head to 0

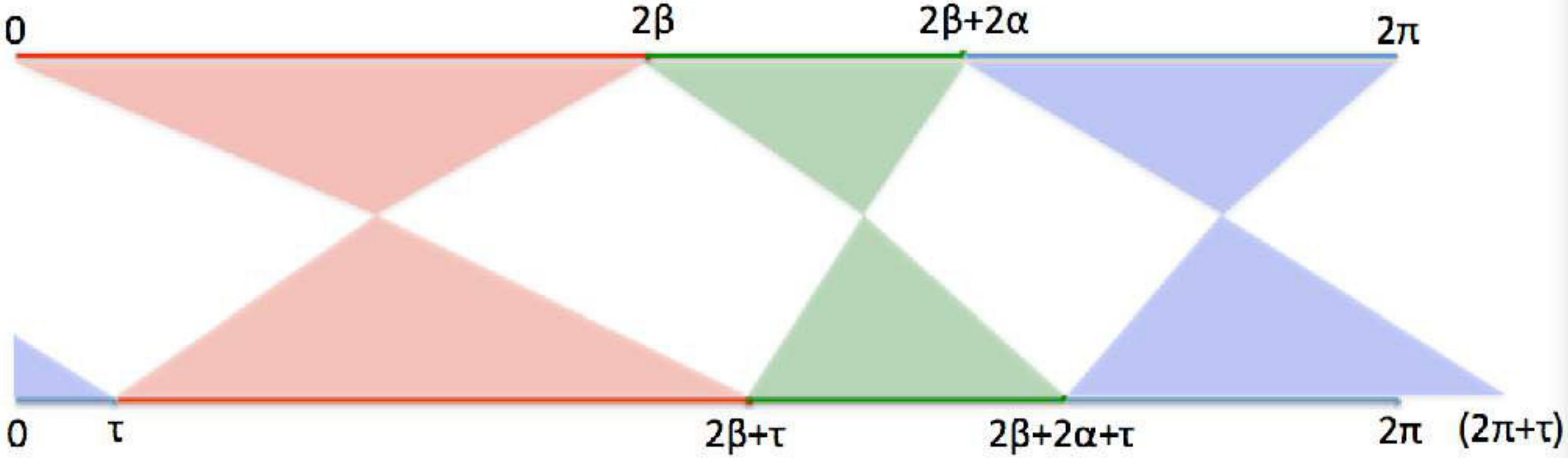


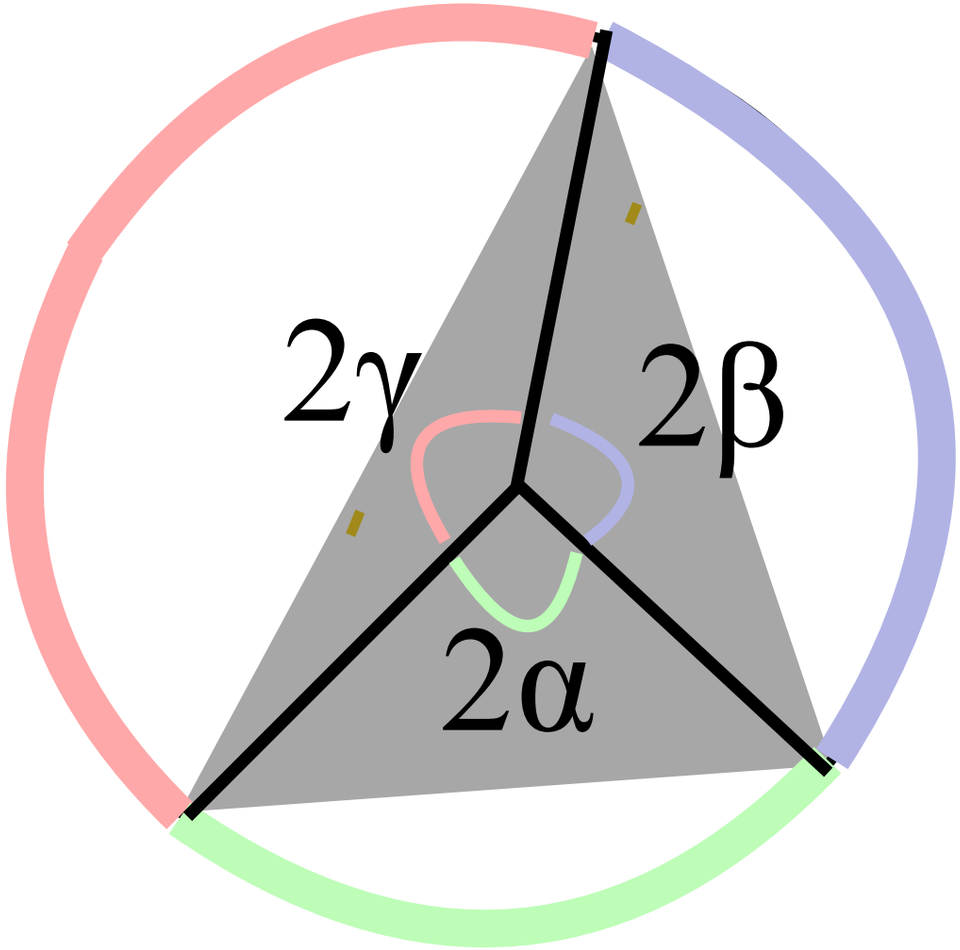
Our Interval Exchange Transformation (IET) is defined by:

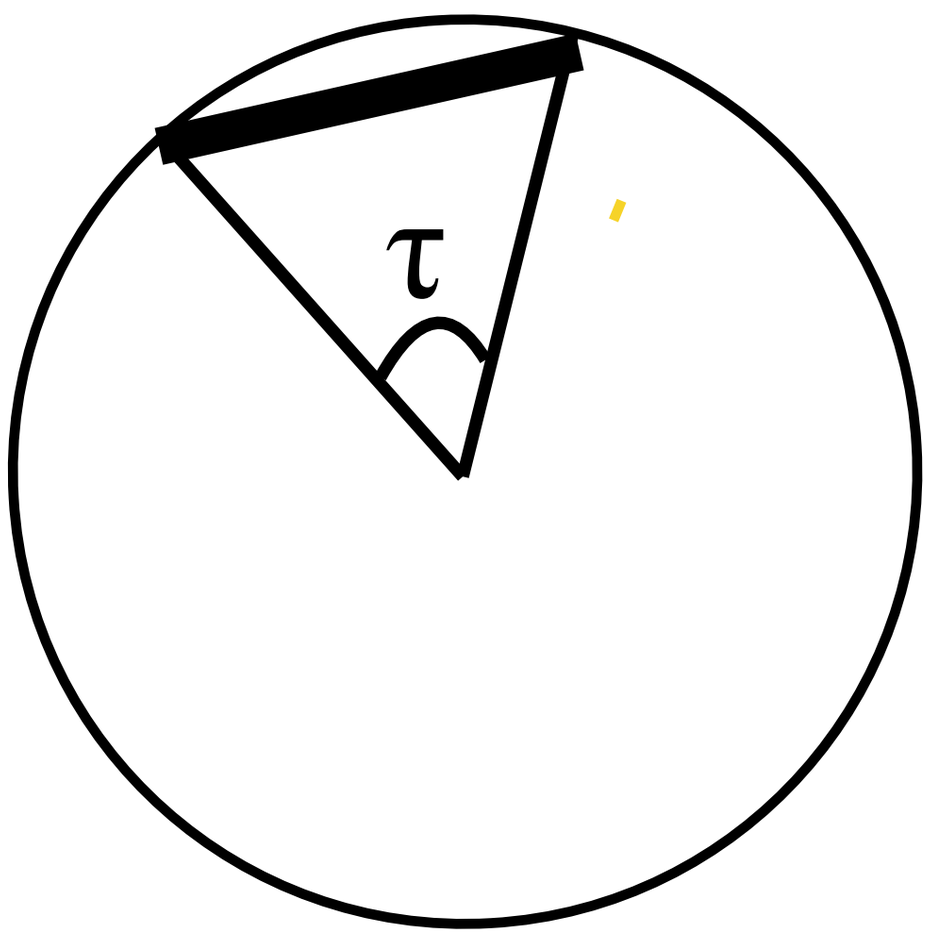
$$X' = \begin{cases} \tau + 2\beta - X & \text{if } 0 < X < 2\beta \\ \tau + 2\beta - 2\gamma - X & \text{if } 2\beta < X < 2\beta + 2\gamma \\ \tau - 2\gamma - X & \text{if } 2\beta + 2\gamma < X < 2\pi \end{cases}$$

… an orientation-reversing IET.

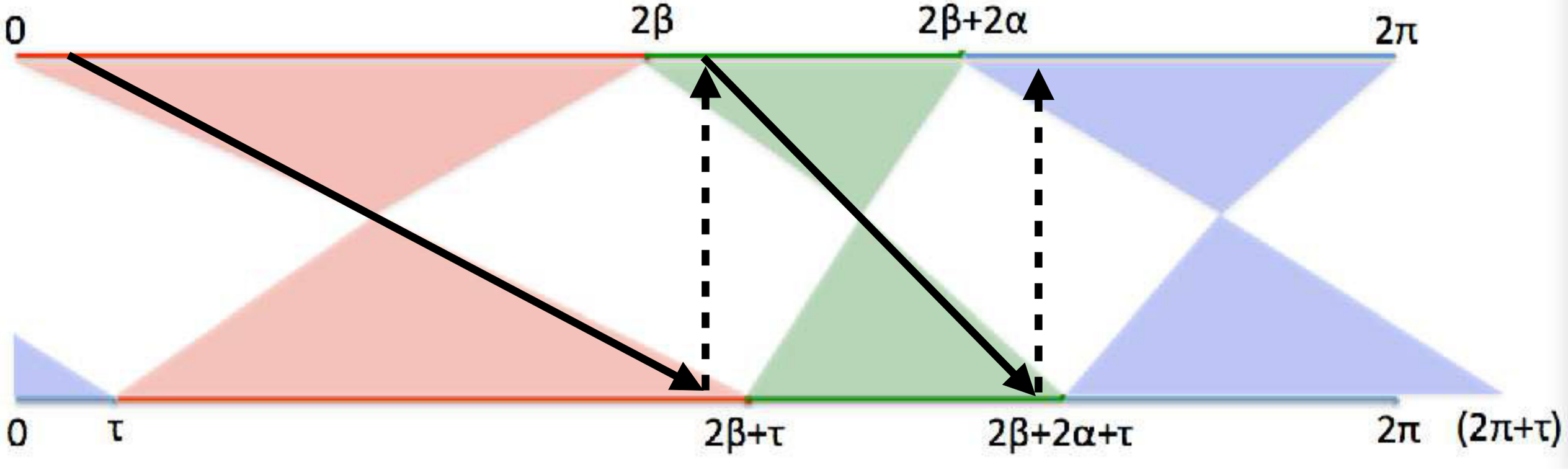
Tiling billiards IET





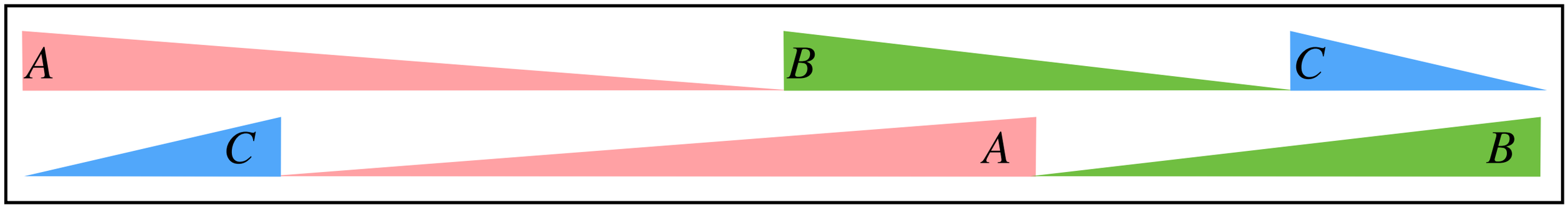


Tiling billiards IET

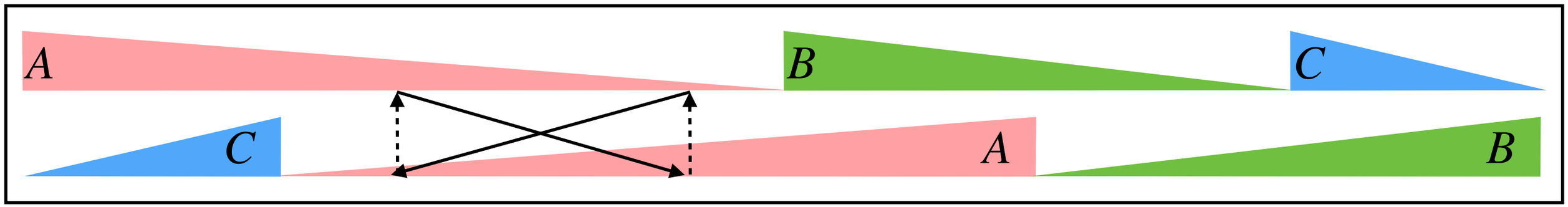


Interval **lengths**: angles
Shifts: angles & trajectory
Starting **point**: orientation of triangle

A short advertisement for flipped IETs



A short advertisement for flipped IETs



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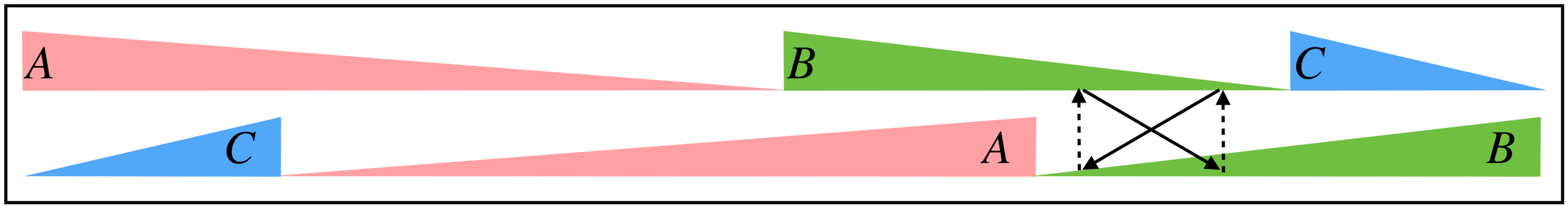

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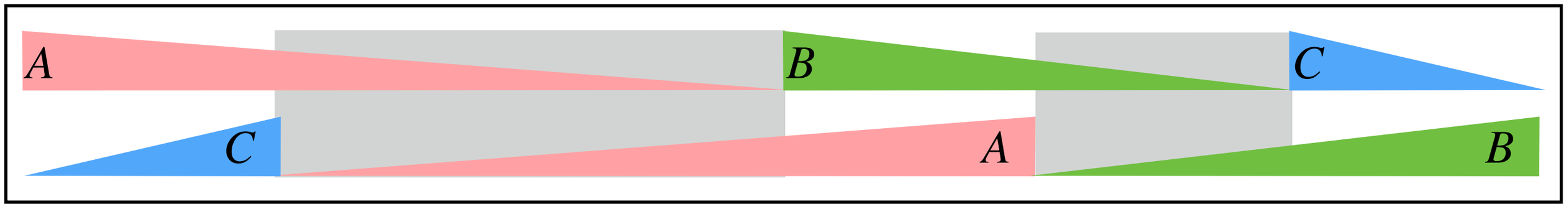
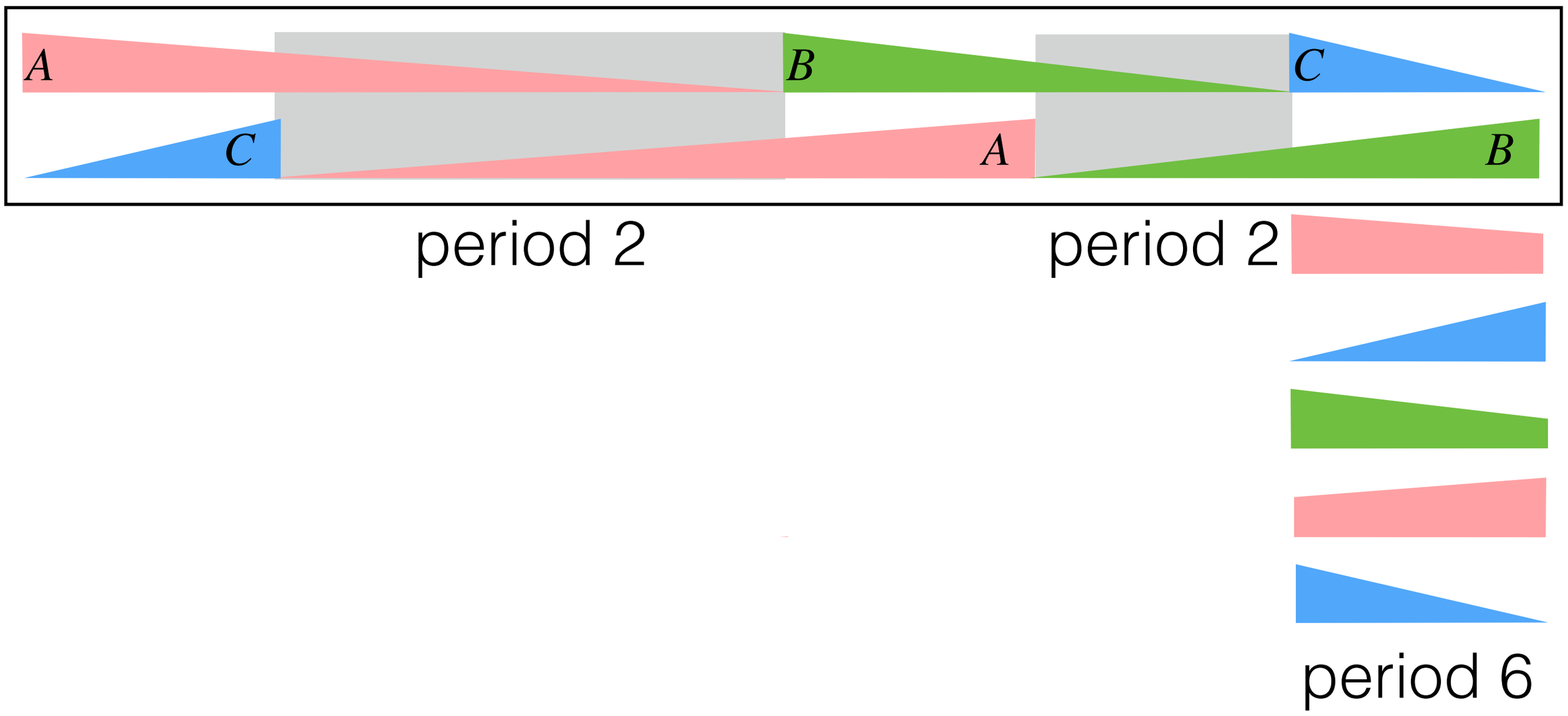
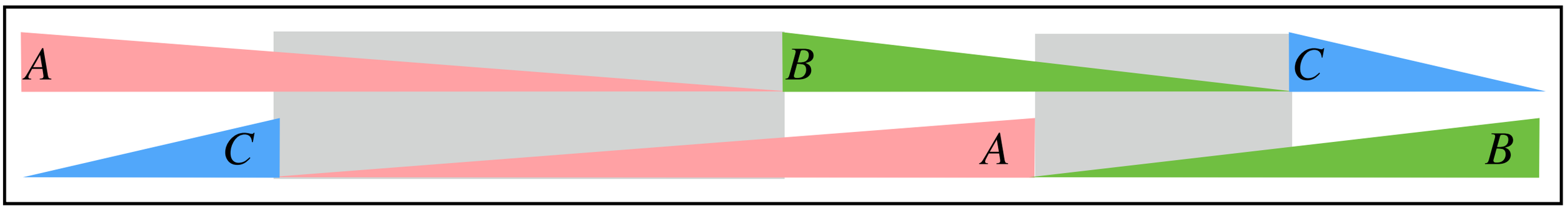


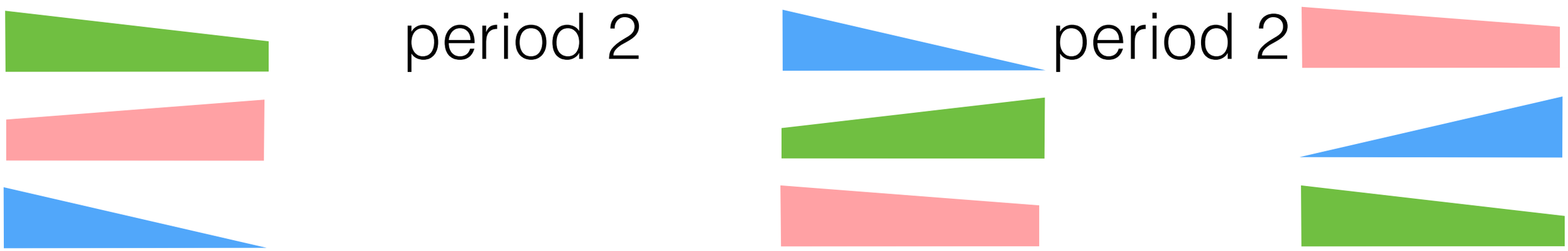
period 2 period 2

A short advertisement for flipped IETs



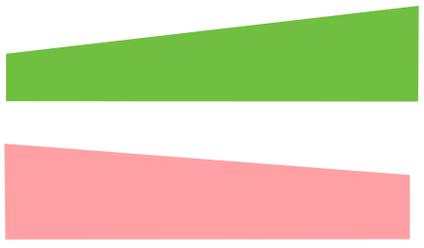
A short advertisement for flipped IETs



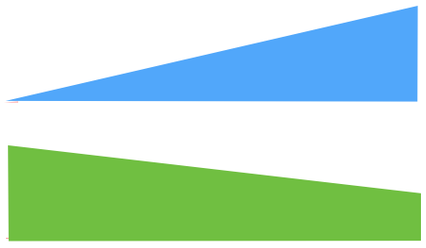


period 2

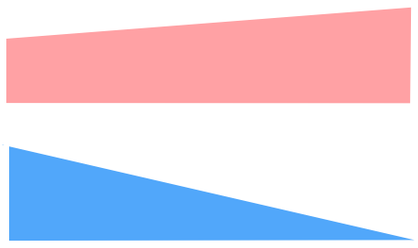
period 2



period 6



period 6



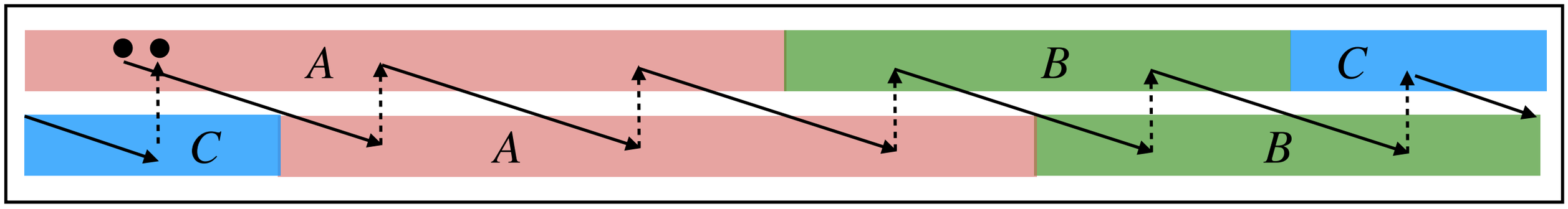
period 6

Everything is flipped periodic, every point is stable, periods of form 4n+2

Comparison to non-flipped IETs

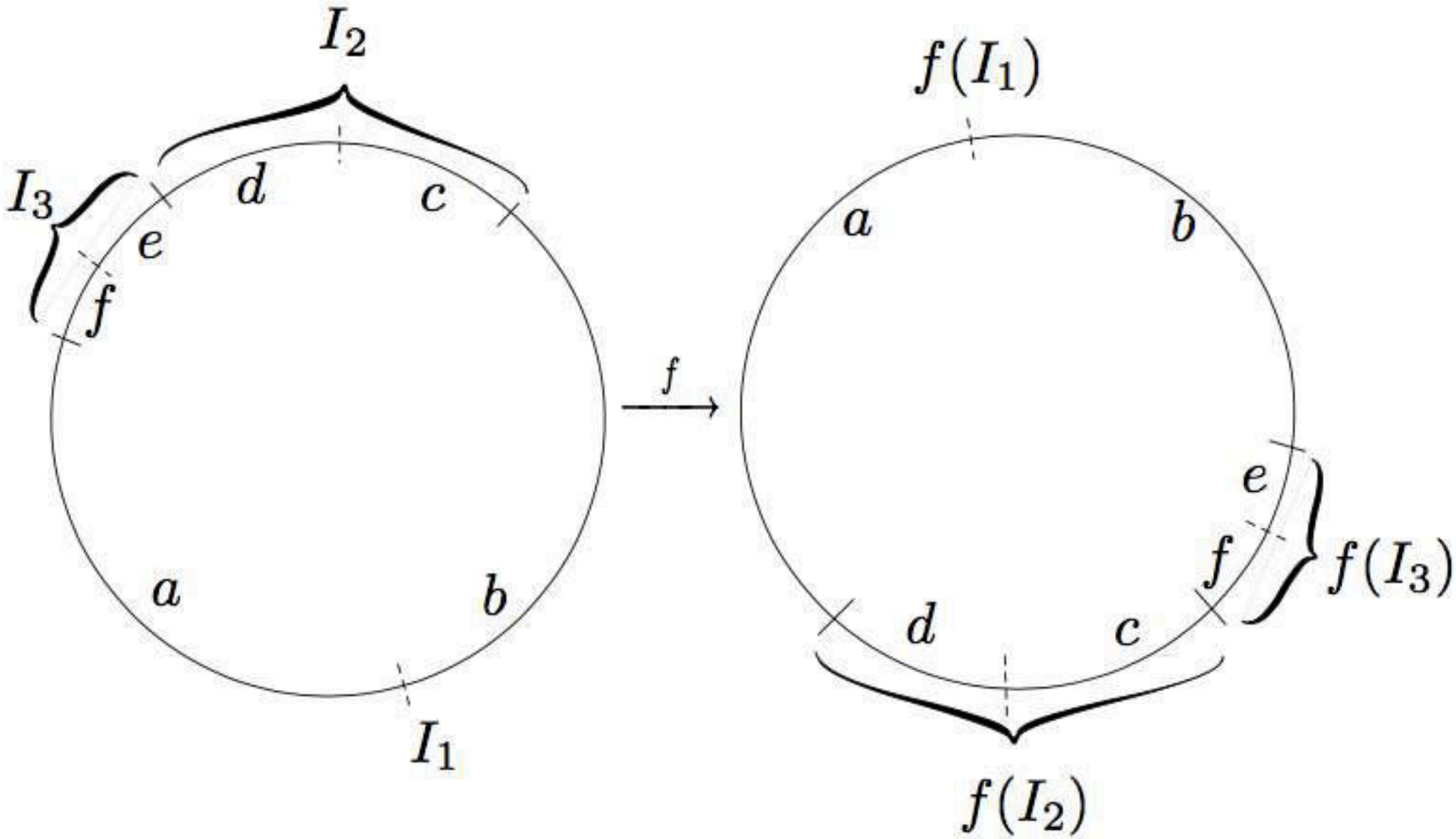
If |AB| and |C| are irrationally related, every point is aperiodic.

Comparison to non-flipped IETs



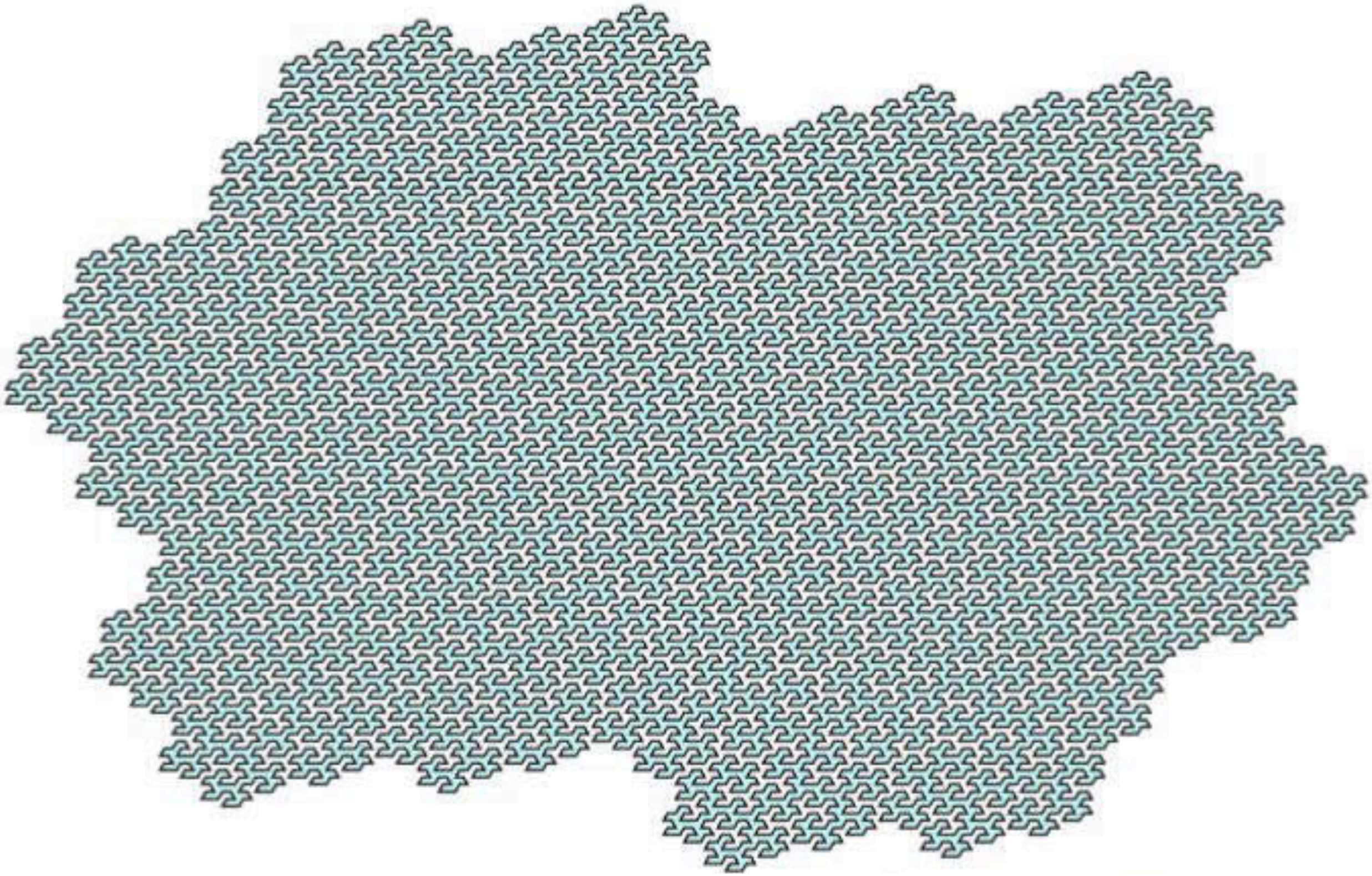
If |AB| and |C| are irrationally related, every point is aperiodic.

Arnoux-Yoccoz IET

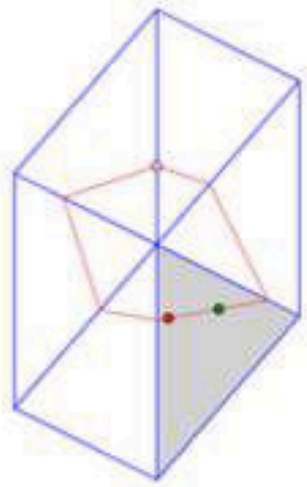
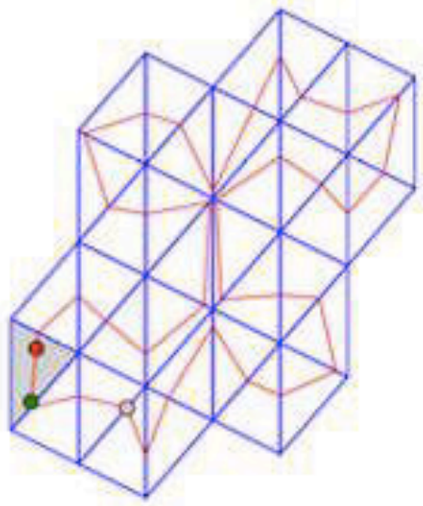
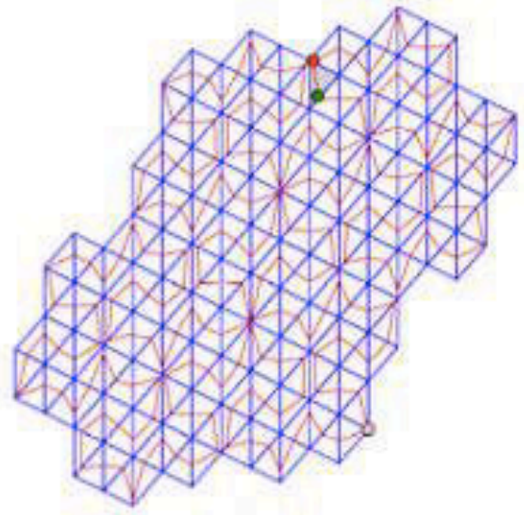
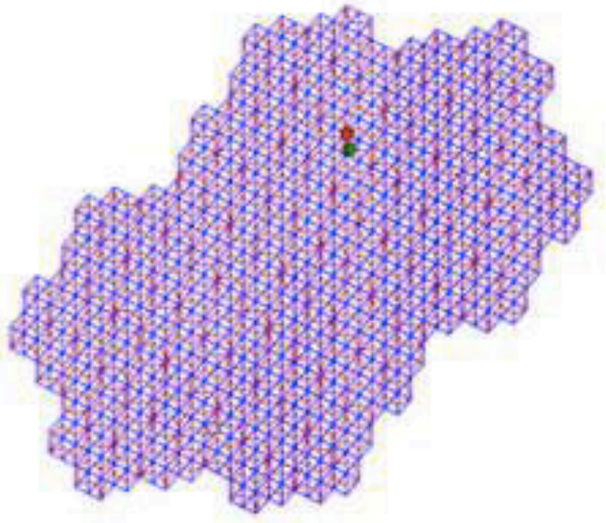
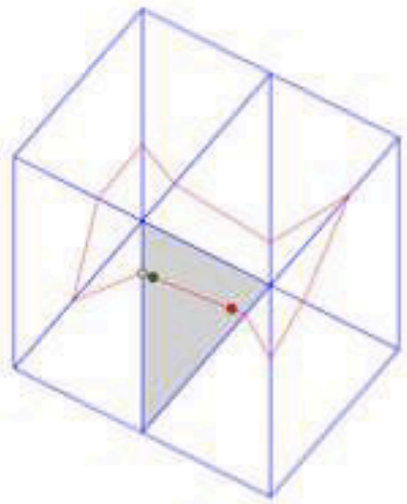
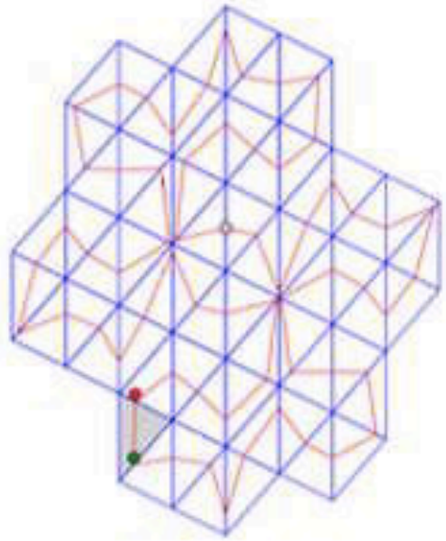
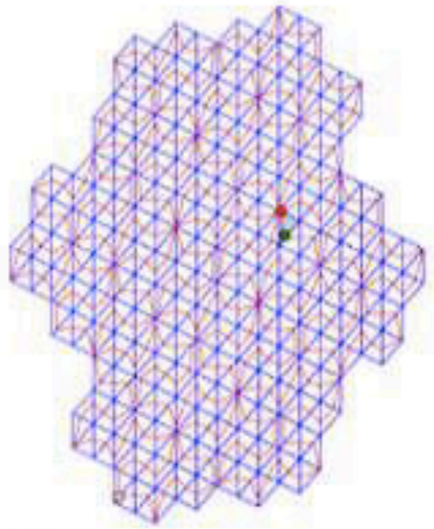
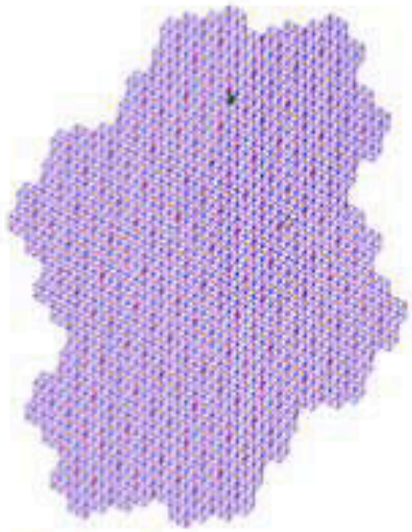
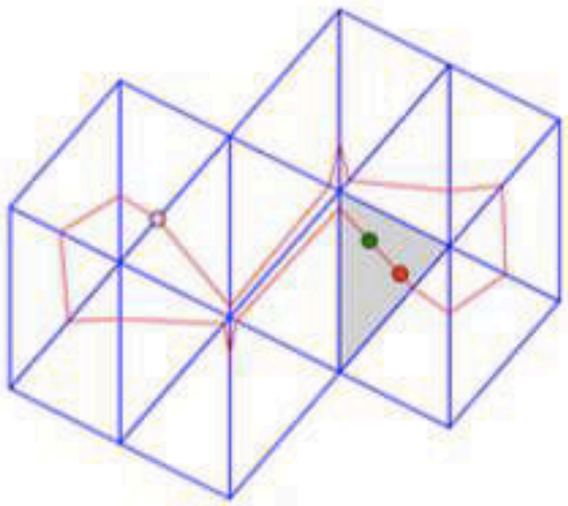
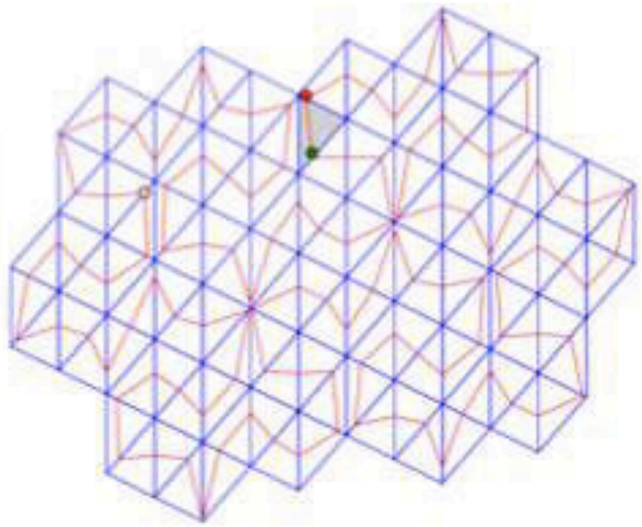
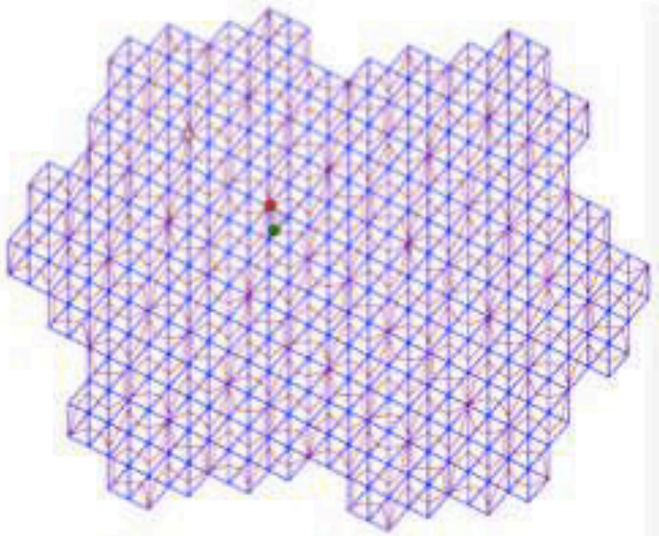
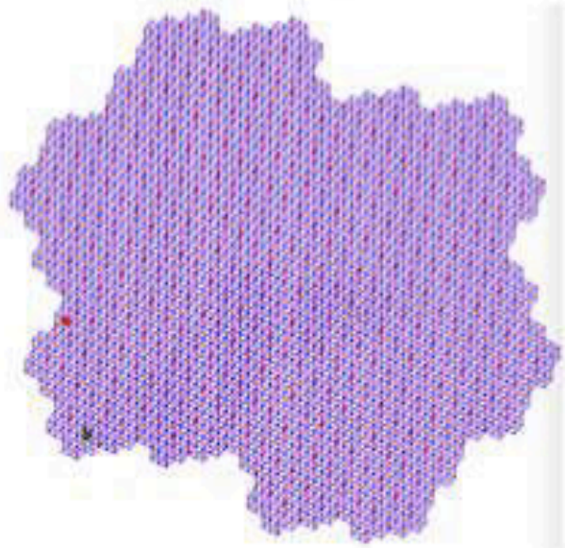


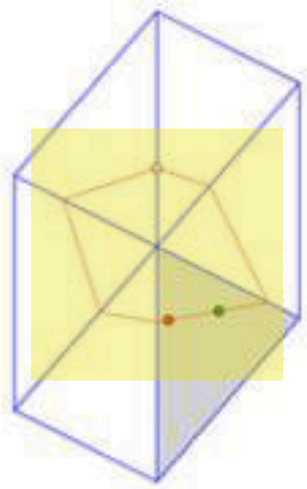
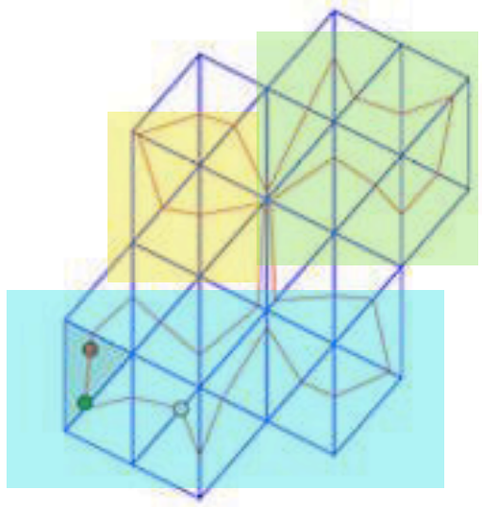
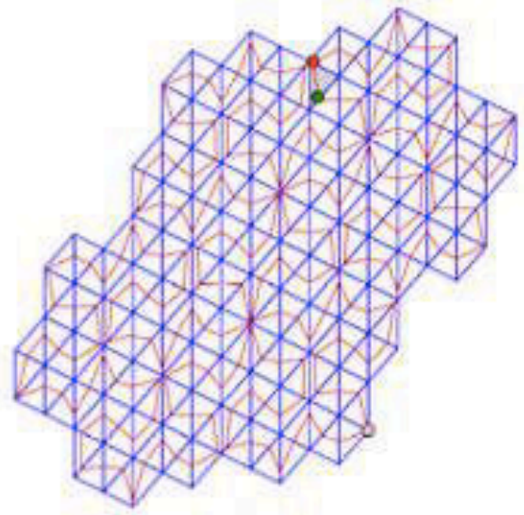
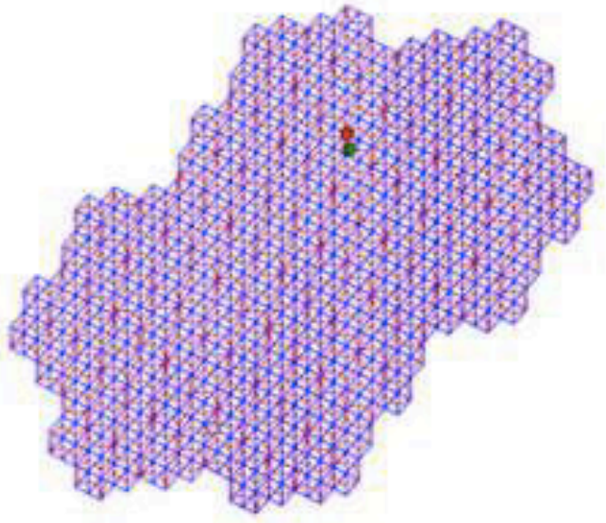
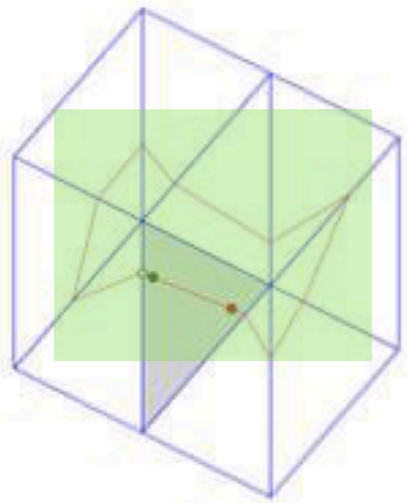
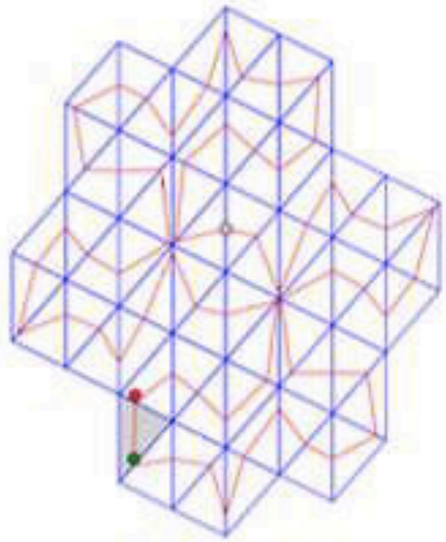
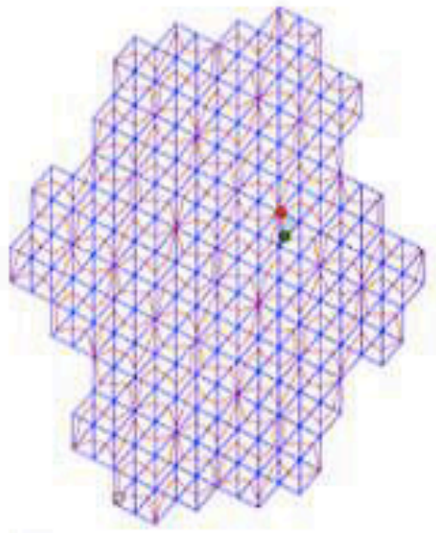
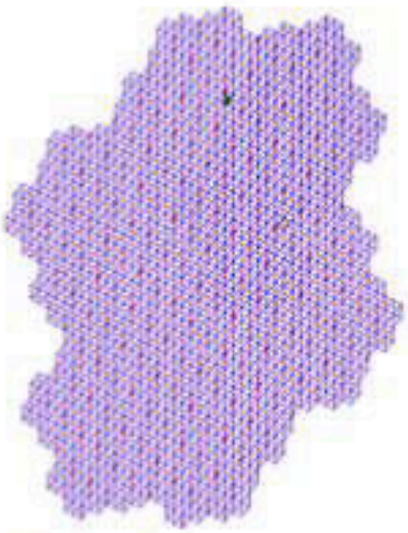
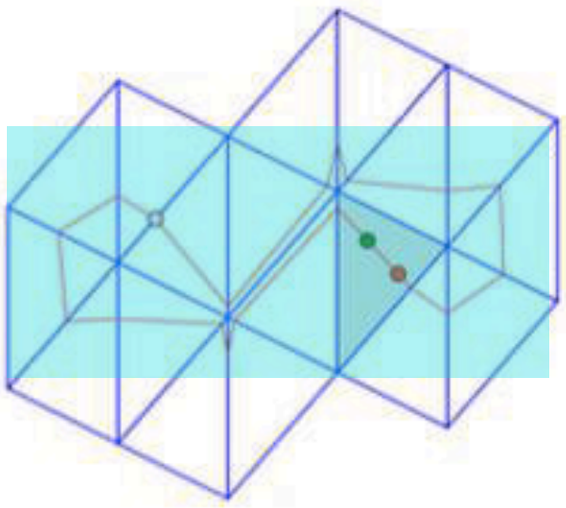
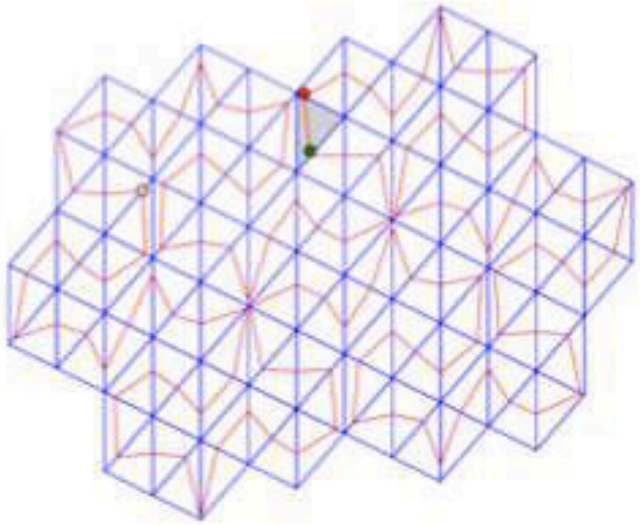
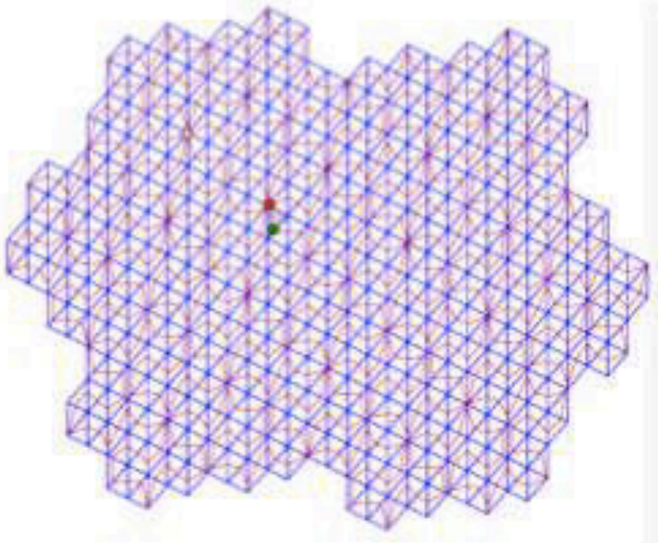
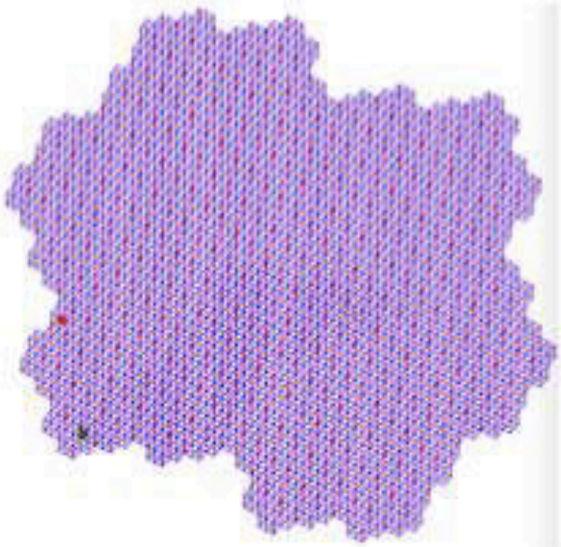
- Switch a&b, c&d, e&f
- Rotate a half turn

Rauzy fractal

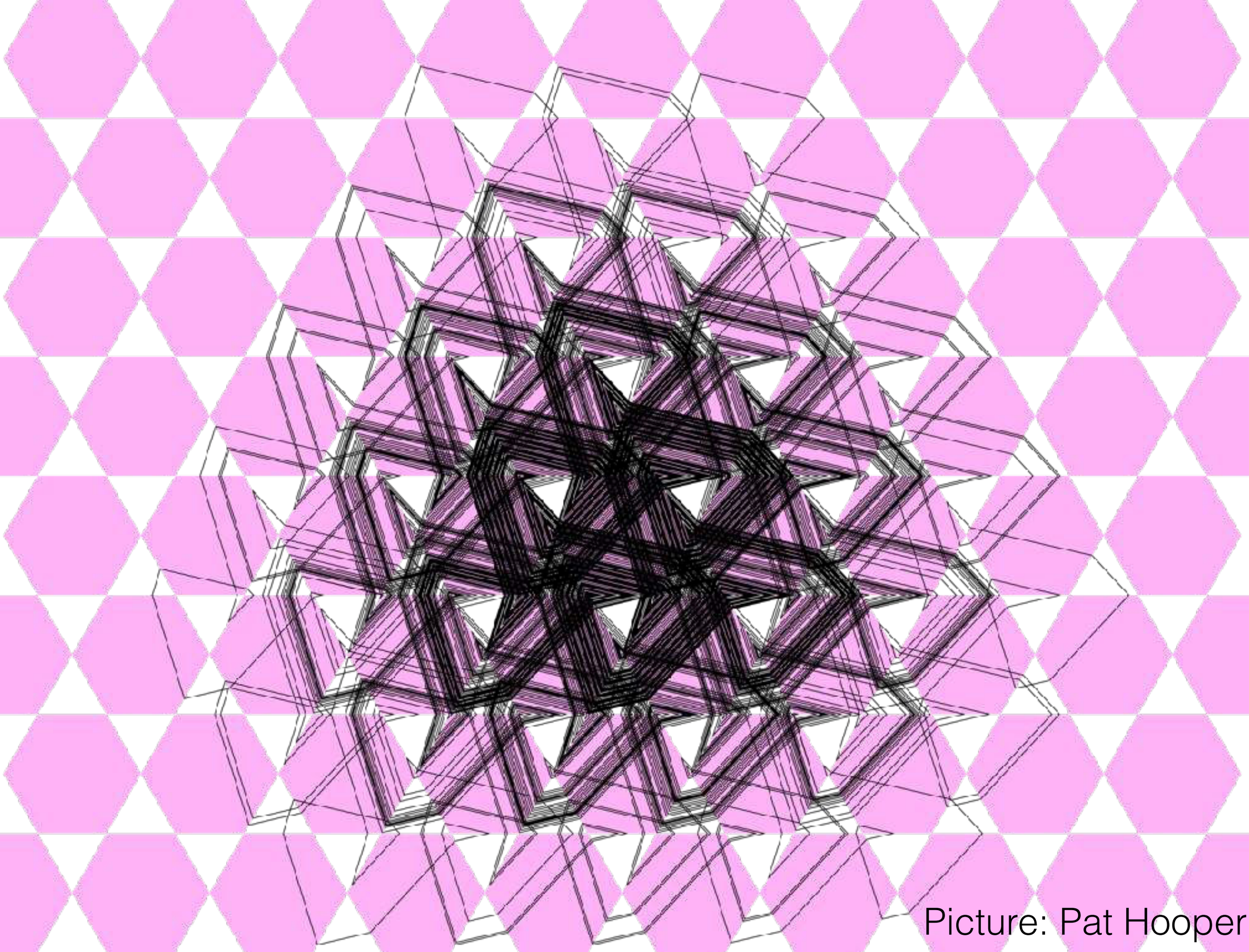


from Hooper & Weiss

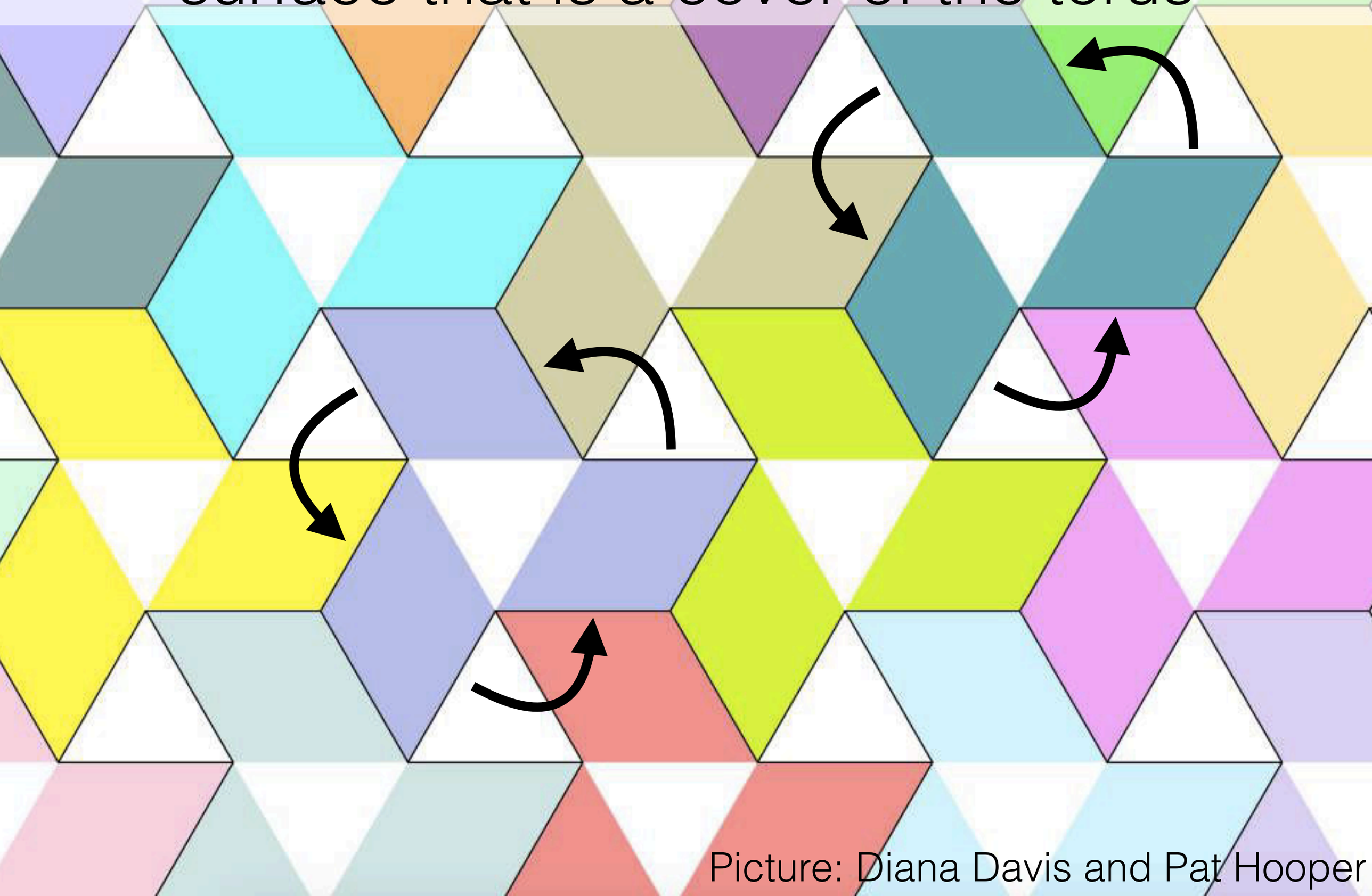













Trihexagonal tiling collaborator



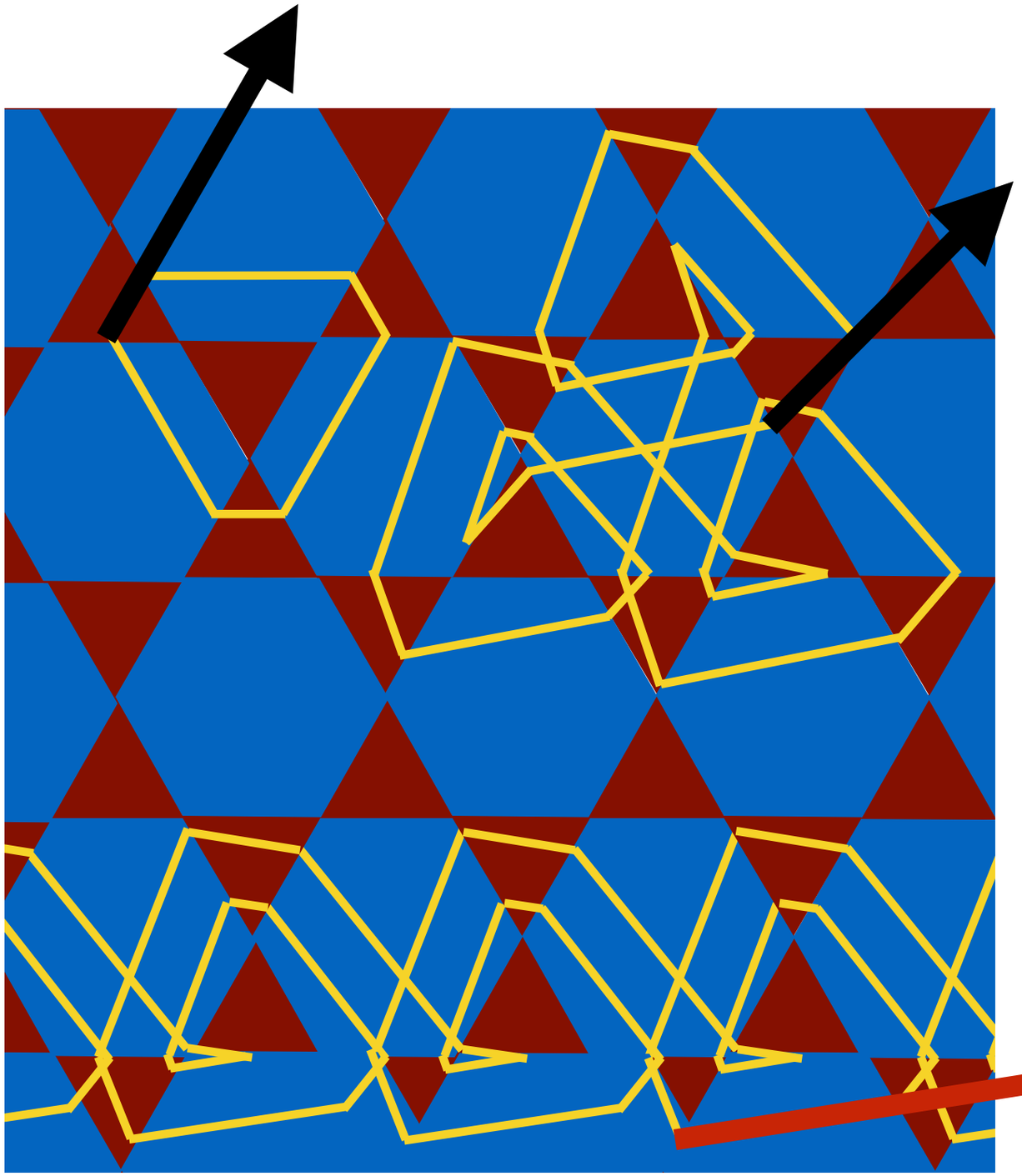
Picture: Pat Hooper

Glue rhombuses to get an infinite translation surface that is a cover of the torus



Picture: Diana Davis and Pat Hooper

Two types of periodic behavior:



Periodic directions

Drift-periodic directions

Theorem: **Periodic** and **drift-periodic** directions are those in the triangle grid.