

## 1 My work as an educator

I have spent many years developing my craft both as a mathematician and an educator. I have put serious work and thought into the best ways to learn, and thus the best ways to teach, mathematics. I will discuss various facets of my educational philosophy and practice below, but let me start by giving you some facts about my experience.

- I have taught for a total of 10 years at Brown University, Northwestern University, Williams College and Swarthmore College.
- I have taught courses across the entire undergraduate curriculum – combinatorial game theory and a course on prime numbers for non-majors, all levels of calculus, linear algebra, discrete mathematics, introduction to proofs, real analysis, applied real analysis, and a senior seminar on surfaces and billiards.
- I have advised student mathematics research for five summers, resulting in three papers, and I have applied to be the director of an REU in 2021; see § 8–10. Six of the eight undergraduates I have mentored are now in Ph.D. programs.
- My [video explaining my Ph.D. thesis result](#) won its category in the “Dance your Ph.D.” competition, and has been viewed over 100,000 times.
- I enjoy giving talks, and I’ve given over 100 of them now. My talks combine serious mathematics, beautiful pictures, and stand-up comedy.

## 2 How I like to run classes

Students have nightly homework, which consists of about 10 interesting problems that I have written, which build and explore the course material. During class, students discuss their solutions in groups of five.



I like to hold classes outside, especially on the day when the flowering trees explode into color.

I have two powerful *incentives* in place, which motivate students to actively discuss mathematics:

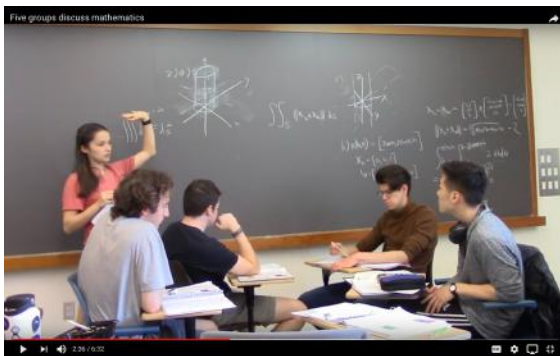
- I check each student's notebook to make sure they have worked on each problem enough to be prepared for a productive discussion, and
- after class, they write up their solutions to turn in for a grade.

I have taught both lecture-based and discussion-based courses, with class sizes of 12, 16, 25 and 30. My experience has been that, as long as the room's blackboard and furniture setup is sufficient, larger classes actually work *better*. I am eager to try discussion-based courses with larger class sizes, whenever the opportunity may arise.

### 3 My classes in action

Everywhere I work, I try to have each of my colleagues sit in on my classes at least once, and I try to sit in on theirs at least once, as well. Since you will likely not be able to attend my classes (please stop by anytime!), I have posted videos online:

- [Five groups discuss mathematics](#) (six-minute, whole-room panorama)
- [Students discussing under a tree](#) (a three-minute clip of one group's discussion)



Screenshots from “Five groups discuss mathematics” and “Students discussing under a tree,” videos of my multivariable calculus course in spring 2019

If you sit in on my class or if you watch one of these videos, you will not see me doing anything particularly amazing. You will find that it is the *students* who are doing amazing things! Usually, my colleagues' first question at the end of observing my class is, “How did you get the students to do that?” My job is to write engaging problems, and to create a classroom atmosphere of kindness, curiosity and learning.

## 4 Teaching awards

- Northwestern University Dept of Mathematics, award for Excellence in Teaching
- Brown University, nominated for Presidential Award for Excellence in Teaching
- Brown University, Mathematics Department Outstanding Teaching Award
- Williams College, Morgan prize for teaching (as an undergraduate TA)

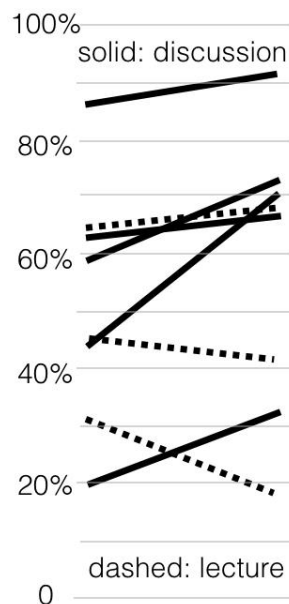
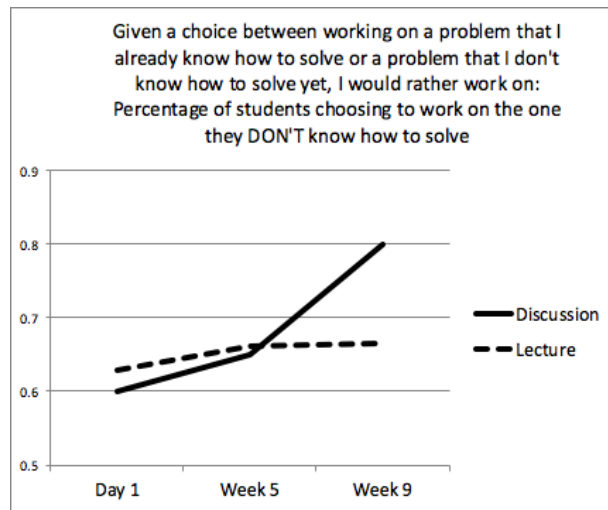
## 5 Some data on students' growth mindset

My goal is to teach students to enjoy working on challenging problems, a trait that is often called *growth mindset*.

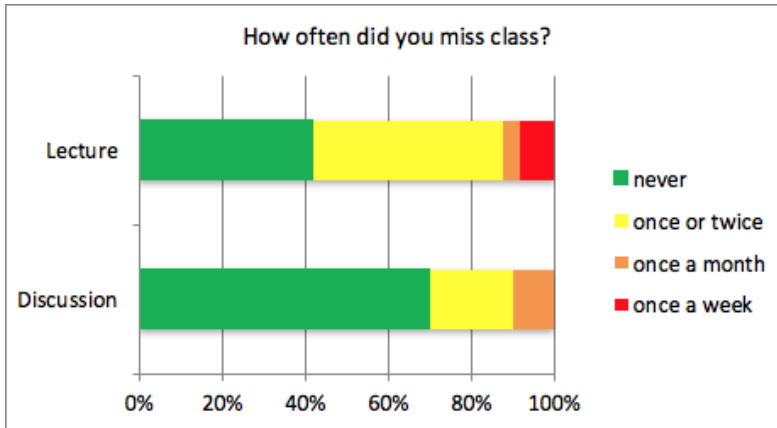
When I was the course head for honors multivariable calculus and linear algebra at Northwestern, I decided to replace the weekly “recitation section” with a problem-solving session. The math department leadership liked this idea enough to try it in all of the calculus courses – a change affecting approximately 1000 students – the following year.

I had the opportunity to teach one section of the Northwestern course in an entirely discussion-based method. The other instructors (who were teaching lecture-based courses as usual) and I collected data throughout the semester to measure, among other things, students' growth mindset. The results (in top picture) show that, at the beginning of the quarter, 6/10 of the students in my discussion-based course section said they would prefer to work on a problem that they didn't know how to solve, and by the end of the quarter, it had increased to 8/10 (see chart above). By contrast, the percentage in the lecture-based sections did not change much.

This is no commentary on my colleagues at Northwestern, who are all exceptional teachers! Indeed, I have since replicated this finding in all of my own lecture-based and discussion-based courses (see picture to the right). For discussion of statistical significance and much more data, see my paper [Inquiry-based learning in a first-year honors course](#) [D18].



## 6 Actively engaging students during class



I prefer to teach discussion-based classes because, while students do listen attentively in lectures, they have such rich mathematical conversations with their classmates when they have the opportunity to have discussions, that I like to give them the opportunity to do that in class. Students also

tend to show up more often (see chart above, from [D18]), and I like to see them.

## 7 The problem sets

All of my problem-based curriculum materials are freely available, and everyone is welcome to use them. You may wish to read through them, to see how the problems on each page build on previous problems, and how each night's homework has a mix of several topics. The problem books are linked below.

- [Calculus 1](#) (Swarthmore)
- [Discrete Mathematics](#) (Swarthmore)
- [Multivariable Calculus](#) (Swarthmore)
- [Introduction to Proofs](#) (Northwestern)
- [Real Analysis](#) (Swarthmore)
- [Billiards, Surfaces and Geometry](#) (Williams)

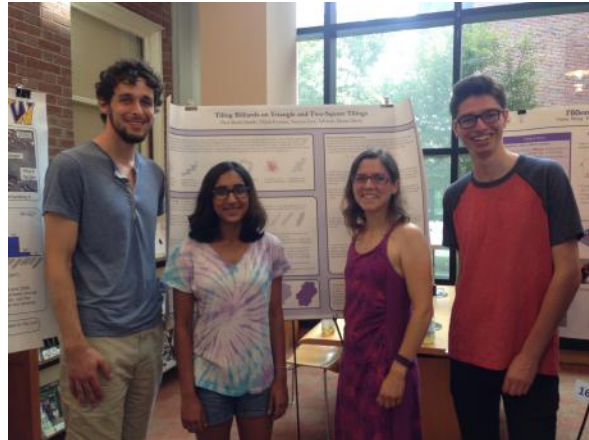
All of these are living documents: I re-order and re-word the problems every time I teach the course, and each day in class I note down ways that I could improve the problems the next time, either to be clearer or to ask a better question.

My problem sets are currently in use at Brooklyn College (billiards), Wright State University (analysis), Phillips Exeter Academy (discrete math; PEA is the original source of many of the problems), and the Solebury School (multivariable calculus).

## 8 Undergraduate research

I have mentored student research for five summers and one academic year:

- Summer@ICERM REU at Brown in 2012 and 2013,
- SMALL REU at Williams in 2016,
- three senior thesis students at Williams in 2016–2017,
- a “billiards cluster” research group at Tufts in 2017, and
- five high school students at the PROMYS program at Boston University in 2019.



I enjoy working closely with students, helping them to find questions that excite them, encouraging them to explore and discover, and challenging them to write down their ideas rigorously. Of all the students I have mentored, half are female and half are male, and about half are members of underrepresented minority groups.

My research students have worked in several areas:

- My three REU groups all worked on the same “tiling billiards” problem (see my research statement for details). The 2012 group explored basic ideas, the 2013 group proved 2012’s conjectures and extended results to more tilings, and the 2016 group connected the problem to interval exchange transformations (IETs), a large area of dynamical systems research.
- The paper from the 2013 group [DDRS18] has been published in *Advances in Geometry*, and has already been cited several times and extended. The paper from the 2016 group [BDF18] has been accepted pending revision to *Bulletin of the London Mathematical Society* and others have proved our conjectures and fully developed the theory [HuP18, P19]. The paper from the 2017 cluster [CCDLO18] has been submitted for publication.
- My three senior thesis students worked in very different areas: billiards, minimal surfaces, and systems of cranks. All three proved original theorems.
- The high school students at BU ran simulations of toy versions of gerrymandering problems and showed that clustering benefits the minority.

## 9 Future undergraduate research

I have many undergraduate projects that I would like to have students work on in the future; see my research statement for details. I have applied to be the director of Summer@ICERM in 2021, where about 20 students will use mathematics to work on gerrymandering problems related to the redistricting that will follow the 2020 Census. This will extend the work of the students I mentored in 2019.



## 10 Where my students are now

Student		Current program
Kelsey DiPietro	Summer@ICERM 2013	Ph.D., Notre Dame, applied math
J.T. Rustad	Summer@ICERM 2013	Ph.D., Maryland, mathematics
Alex St Laurent	Summer@ICERM 2013	Bloomberg LP, software engineer
Elijah Fromm	SMALL 2016	Ph.D., Yale, mathematics
Sumun Iyer	SMALL 2016	Ph.D., Cornell, mathematics
Paul Baird-Smith	SMALL '16 & Thesis '16-'17	Ph.D., Texas, computer science
Megumi Asada	Thesis 2016–2017	M.Phil, Cambridge, criminology
Dylanger Pittman	Thesis 2016–2017	Ph.D, Emory, mathematics

## 11 Workshops for other educators

I have given several courses for high school teachers: 10-hour workshops on how to write a problem-centered curriculum, and teach a discussion-based course. Here is some of the feedback that the teachers gave about my June 2019 courses.

### *Writing a problem-centered curriculum*

Diana did a masterful job sharing her process and philosophy for [problem-based learning]. I came to the conference very skeptical of the idea, and am leaving a convert. Diana is easily one of the best facilitators of adult learning I have met.

Diana is a machine. She is probably 25 years my junior, but I could learn about math, teaching, and teaching math all day long from her. She challenged us to know our curriculum, then really know our curriculum. At times I was overwhelmed, but Diana brought out the best in us. The course was so well prepared and so challenging.

I feel like a completely different teacher with a transformed idea about problem/discussion based classes. I couldn't sleep at night because I was so excited.

### *Teaching a discussion-based math course*

Diana has taught me much about the classroom-based discussion strategy in the short time of 5 days, and I feel incredibly empowered by her expertise. She has a vast amount of knowledge and passion for the way she teaches, and I was able to FEEL what it was like to be a student in her class. Amazing experience! I was blown away by how quickly the class went each and every day. Two hours seemed like 30 minutes – that is how engaged we were!

I have also written a [guide for how to write a problem-based curriculum and teach a discussion-based math course](#), which I hope to publish soon as a preface to my problem-based billiards curriculum.

## Outreach: mathematical jewelry

For the past two years, I have been making laser-cut earrings based on my research (Figure 3): they are pentagonal, and decorated with periodic billiard trajectories on the regular pentagon that I discovered (see my research statement for details). I have given away hundreds of pairs of earrings – at math conferences, to the people next to me on the airplane, to my friends and family, to students, to students’ parents – to almost literally *everyone* I meet. The Museum of Mathematics in New York City also sells them in their gift shop. These earrings inspire universal delight.



Figure 3: Examples of periodic billiard trajectory earrings that I have made out of solid basswood (left two pictures) and plastic (right two pictures).

Even people who claim not to like math see them and immediately start asking about what they are. How many situations are there, where someone who was not even interested in mathematics a minute ago is suddenly asking about your *research*? I feel very fortunate to be able to create that spark of curiosity in people’s minds.

I am currently learning to 3D print the earrings instead of laser cutting them; the “Illustrating Mathematics” semester program at ICERM that I am attending in Fall 2019 is a perfect place to do this, as there are experts on all aspects of “making,” and resources to try new techniques. I have been giving the earrings away for free to children and students, and I sold them at a booth in the Exhibition Hall at JMM in 2019. I will do the same in 2020 in Denver, so if you are at JMM, please come visit.

## 12 Working towards a diverse mathematical world

When I taught multivariable calculus in Spring 2018, I gave a “mathematician spotlight” at the beginning of each class:

- For two minutes, I showed a picture of someone who is currently doing mathematics, gave a brief bio, and explained something about the person’s research.
- I alternated male and female mathematicians,<sup>1</sup> with half white and half non-white, and about a quarter LGBTQ.
- On their surveys at the end of the course, several students said that the mathematician spotlight was their favorite part of the course, the part that they would remember the most.
- On the same surveys, I asked the students if they had noticed any patterns among the mathematicians I chose to spotlight. Various students noted that many were women, or people of color, or were working in dynamical systems – but a significant number wrote “didn’t notice any patterns,” or something similar.

To my students, then, this world I created for them where everyone has equal access to mathematics seemed totally normal! I find this very inspiring, and I plan to continue to work to create such a world in the future.

## Moving forward

I like building things: building community within my classroom and with colleagues in the department, and building engaging curriculum that improves year over year. I am excited to spend my career working each day on building and things alongside you, and learning everything that you and your institution can teach me.

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<sup>1</sup>I didn’t find any nonbinary mathematicians. I did feature several trans mathematicians.



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## References

- [BDFI18] Paul Baird-Smith, Diana Davis, Elijah Fromm and Sumun Iyer, *Tiling billiards on triangle tilings*, arxiv:1809.07876, preprint (2018).
- [CCDLO18] Aaron Calderon, Solly Coles, Diana Davis, Justin Lanier and André Oliveira, *How to hear the shape of a billiard table*, submitted (2018).
- [D18] Diana Davis, *Inquiry-based learning in a first-year honors course*, PRIMUS, 28(5), 387–408 (2018).
- [DDRS18] Diana Davis, Kelsey DiPietro, Jenny Rustad, and Alexander St Laurent, *Negative refraction and tiling billiards*, Advances in Geometry, 18(2), 133–159 (2018).
- [HuP18] Pascal Hubert, Olga Paris-Romaskevich, *Triangle tiling billiards draw fractals only if aimed at the circumcenter*, preprint (2018). arXiv:1804.00181.
- [P19] Olga Paris-Romaskevich, *Trees and flowers on a billiard table*, preprint (2019), arXiv:1907.01178.